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FUZZY q-IDEALS OF BCI-ALGEBRAS WITH DEGREES IN THE INTERVAL $(0,1]$

Se Kyung Sung* and Sun Shin Ahn**

Abstract. The notion of an enlarged q-ideal and a fuzzy q-ideal in BCI-algebras with degree are introduced. Related properties of them are investigated.

1. Introduction

The concept of a fuzzy set is applied to generalize some of the basic concepts of general topology([1]). Rosenfeld([7]) constituted a similar application to the elementary theory of groupoids and groups. Xi([8]) applied to the concept of fuzzy set to BCK-algebras. Y. L. Liu et al.([6]) defined the notions of q-ideals and a-ideals in BCI-algebras and studied their properties.

In this paper, we introduce the notion of an enlarged q-ideal and a fuzzy q-ideal in BCI-algebras with degree. We study related properties of them.

2. Preliminaries

We review some definitions and properties that will be useful in our results.

By a BCI-*algebra* we mean an algebra $(X;*,0)$ of type (2,0) satisfying the following conditions:

(a1) $(\forall x, y, z \in X)\ (((x*y)*(x*z))*(z*y)=0)$,
(a2) $(\forall x, y \in X)\ ((x*(x*y))*y=0)$,
(a3) $(\forall x \in X)\ (x*x=0)$,
(a4) $(\forall x, y \in X)\ (x*y=0,\ y*x=0 \Rightarrow x=y)$.

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Correspondence should be addressed to Sun Shin Ahn, sunshine@dongguk.edu.

A BCI-algebra X is called a *BCK-algebra* if it satisfies the following identity:

(a5) $(\forall x \in X)\ (0 * x) = 0$.

In any BCI-algebra X one can define a partial order "$\le$" by putting $x \le y$ if and only if $x * y = 0$.

A BCI-algebra X has the following properties:

(b1) $(\forall x \in X)\ (x * 0 = x)$.
(b2) $(\forall x, y, z \in X)\ ((x * y) * z = (x * z) * y)$.
(b3) $(\forall x, y \in X)\ (0 * (x * y) = (0 * x) * (0 * y))$.
(b4) $(\forall x, y \in X)\ (x * (x * (x * y)) = x * y)$.
(b5) $(\forall x, y, z \in X)\ (x \le y \Rightarrow x * z \le y * z,\ z * y \le z * x)$.
(b6) $(\forall x, y, z \in X)\ ((x * z) * (y * z) \le x * y)$.
(b7) $(\forall x, y, z \in X)\ (0 * (0 * ((x * z) * (y * z))) = (0 * y) * (0 * x))$.
(b8) $(\forall x, y \in X)\ (0 * (0 * (x * y)) = (0 * y) * (0 * x))$.

A non-empty subset S of a BCI-algebra X is called a *subalgebra* of X if $x * y \in S$ whenever $x, y \in S$. A non-empty subset A of a BCI-algebra X is called an *ideal* of X if it satisfies:

(c1) $0 \in A$,
(c2) $(\forall x \in A)\ (\forall y \in X)\ (y * x \in A \Rightarrow y \in A)$.

Note that every ideal A of a BCI-algebra X satisfies:

$$(\forall x \in A)\,(\forall y \in X)\,(y \le x \Rightarrow y \in A).$$

A non-empty subset A of a BCI-algebra X is called a *q-ideal*([6]) of X if it satisfies (c1) and

(c3) $(\forall x, y, z \in X)(x * (y * z) \in A \text{ and } y \in A \Rightarrow x * z \in A)$.

Note that any q-ideal is an ideal, but the converse is not true in general.

We refer the reader to the book [2] for further information regarding BCI-algebras.

A fuzzy subset μ of a BCK/BCI-algebra X is called a *fuzzy ideal*([4]) of X if it satisfies:

(d1) $(\forall x \in X)(\mu(0) \ge \mu(x))$,
(d2) $(\forall x, y \in X)(\mu(x) \ge \min\{\mu(x * y), \mu(y)\})$.

PROPOSITION 2.1. *If μ is a fuzzy ideal of a BCI-algebra X, then the following holds:*

$$(\forall x, y \in X)(x \le y \Rightarrow \mu(x) \ge \mu(y)).$$

Proof. Straightforward. □

3. Fuzzy q-ideals of BCI-algebras

DEFINITION 3.1. A fuzzy subset μ of a BCI-algebra X is called a *fuzzy q-ideal* of X if it satisfies (d1) and

(d3) $(\forall x, y, z \in X)(\mu(x * z) \geq \min\{\mu(x * (y * z)), \mu(y)\}$.

EXAMPLE 3.2. Let $X = \{0, a, b, c\}$ be a BCI-algebra([6]) in which the $*$-operation is given by the following table:

$*$	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

Noth that $\{0, a\}$ is a q-ideal of X. Define a fuzzy subset $\mu : X \to [0, 1]$ by

$$\mu = \begin{pmatrix} 0 & a & b & c \\ 0.8 & 0.7 & 0.5 & 0.5 \end{pmatrix}$$

Then μ is a fuzzy q-ideal of X.

PROPOSITION 3.3. *Every fuzzy q-ideal of a BCI-algebra X is a fuzzy ideal of X.*

Proof. Let μ be a fuzzy q-ideal of X. Let $x, y \in X$. Putting $z := 0$ in Definition 3.1(d3) and using (b1), we have

$$\begin{aligned} \mu(x) = \mu(x * 0) &\geq \min\{\mu(x * (y * 0)), \mu(y)\} \\ &= \min\{\mu(x * y), \mu(y)\}. \end{aligned}$$

Hence (d2) holds. Thus μ is a fuzzy ideal of X □

The converse of Proposition 3.3 is not true as seen the following example.

EXAMPLE 3.4. Let $X := \{0, a, b, c\}$ be a BCI-algebra([6]) in which the $*$-operation is given by the following table:

$*$	0	a	b	c
0	0	c	b	a
a	a	0	c	b
b	b	a	0	c
c	c	b	a	0

Note that $\{0\}$ is an ideal of X, but not a q-ideal of X since $c*(0*a) = c*c = 0 \in \{0\}$ and $0 \in \{0\}$ but $c*a = b \notin \{0\}$. Define a fuzzy subset $\mu : X \to [0,1]$ by

$$\mu = \begin{pmatrix} 0 & a & b & c \\ 0.8 & 0.7 & 0.5 & 0.5 \end{pmatrix}$$

Then μ is a fuzzy ideal of X, but not a fuzzy q-ideal of X since $\mu(c*a) = \mu(b) = 0.5 \not\geq 0.8 = \mu(0) = \min\{\mu(c*(0*a)), \mu(0)\}$.

COROLLARY 3.5. *If μ is a fuzzy q-ideal of a BCI-algebra X, then the following holds:*

$$(\forall x, y \in X)(x \leq y \Rightarrow \mu(x) \geq \mu(y)).$$

Proof. It follows from Proposition 2.1 and Proposition 3.3. □

THEOREM 3.6. *If μ is a fuzzy ideal of a BCI-algebra X, then the following are equivalent:*

(1) *μ is a fuzzy q-ideal of X,*
(2) $(\forall x, y \in X)(\mu(x*y) \geq \mu(x*(0*y))$,
(3) $(\forall x, y, z \in X)(\mu((x*y)*z) \geq \mu(x*(y*z))$.

Proof. (1)⇒(2) Let $x, y \in X$. Putting $y := 0$ and $z := y$ in Definition 3.1(d3) and use (d1), we have $\mu(x*y) \geq \min\{\mu(x*(0*y)), \mu(0)\} = \mu(x*(0*y))$. Thus (2) holds.
(2)⇒(3) Since for any $x, y, z \in X$

$$\begin{aligned}((x*y)*(0*z))*(x*(y*z)) =&((x*y)*(x*(y*z)))*(0*z) \\ \leq&((y*z)*y)*(0*z) \\ =&(0*z)*(0*z) = 0,\end{aligned}$$

we have $(x*y)*(0*z) \leq x*(y*z)$. Using (2) and Proposition 2.1, we get

$$\begin{aligned}\mu(x*(y*z)) \leq&\mu((x*y)*(0*z)) \\ \leq&\mu((x*y)*z).\end{aligned}$$

Hence (3) holds.
(3)⇒(1) Let $x, y, z \in X$. Using (d2), (b2), and (3), we have

$$\begin{aligned}\mu(x*z) \geq&\min\{\mu((x*z)*y), \mu(y)\} \\ =&\min\{\mu((x*y)*z), \mu(y)\} \\ \geq&\min\{\mu(x*(y*z)), \mu(y)\}.\end{aligned}$$

Thus μ is a fuzzy q-ideal of X. □

PROPOSITION 3.7. *Let μ be a fuzzy ideal of X. If $\mu(x) \le \mu(x * y)$ for any $x, y \in X$, then μ is a fuzzy q-ideal of X.*

Proof. For any $x, y, z \in X$, we have

$$\begin{aligned}\mu(x * z) \geq & \mu(x) \\ \geq & \min\{\mu(x * (y * z)), \mu(y * z)\} \\ \geq & \min\{\mu(x * (y * z)), \mu(y)\}.\end{aligned}$$

Hence μ is a fuzzy q-ideal of X. □

4. Fuzzy q-ideals of BCI-algebras with degrees in the interval $(0, 1]$

In what follows let X denote a BCI-algebra unless specified otherwise.

DEFINITION 4.1. ([5]) Let I be a non-empty subset of a BCK/BCI-algebra X which is not necessary an ideal of X. We say that a subset J of X is an *enlarged ideal* of X related to I if it satisfies:

(1) I is a subset of J,
(2) $0 \in J$,
(3) $(\forall x \in X)(\forall y \in I)(x * y \in I \Rightarrow x \in J)$.

DEFINITION 4.2. Let I be a non-empty subset of a BCI-algebra X which is not necessary a q-ideal of X. We say that a subset J of X is an *enlarged q-ideal* of X related to I if it satisfies:

(1) I is a subset of J,
(2) $0 \in J$,
(3) $(\forall x, z \in X)(\forall y \in I)(x * (y * z) \in I \Rightarrow x * z \in J)$.

Obviously, every q-ideal is an enlarged q-ideal of X related to itself. Note that there exists an enlarged q-ideal of X related to any non-empty subset I of a BCI-algebra X.

EXAMPLE 4.3. Let $X := \{0, 1, a, b, c\}$ be a BCI-algebra([5]) in which the $*$-operation is given by the following table:

$*$	0	1	a	b	c
0	0	0	a	b	c
1	1	0	a	b	c
a	a	a	0	c	b
b	b	b	c	0	a
c	c	c	b	a	0

Note that $\{0, a\}$ is not both an ideal of X and a q-ideal of X. Then $\{0, 1, a\}$ is an enlarged ideal of X related to $\{0, a\}$ and an enlarged q-ideal of X related to $\{0, a\}$.

Theorem 4.4. *Let I be a non-empty subset of a BCI-algebra X. Every enlarged q-ideal of X related to I is an enlarged ideal of X related to I.*

Proof. Let J be an enlarged q-ideal of X related to I. Putting $z := 0$ in Definition 4.2(3) and use (b1), we have

$$(\forall x \in X)(\forall y \in I)(x * (y * 0) = x * y \in I \Rightarrow x * 0 = x \in J).$$

Hence J is an enlarged ideal of X related to I. □

The converse of Theorem 4.4 does not true in general as seen in the following example.

Example 4.5. Consider a BCI-algebra $X = \{0, a, b, c\}$ as in Example 3.4. Note that $\{0, a\}$ is not both an ideal and a q-ideal of X. Then $\{0, a, b\}$ is an enlarged ideal of X related to $\{0, a\}$ but not an enlarged q-ideal of X related to $\{0, a\}$ since $0 * (a * a) = 0 \in \{0, a\}$ and $0 * a = c \notin \{0, a, b\}$.

In what follows let λ and κ be members of $(0, 1]$, and let n and k denote a natural number and a real number, respectively, such that $k < n$ unless otherwise specified.

Definition 4.6. ([5]) A fuzzy subset μ of a BCK/BCI-algebra X is called a *fuzzy ideal* of X with degree (λ, κ) if it satisfies:

(1) $(\forall x \in X)(\mu(0) \geq \lambda\mu(x))$,
(2) $(\forall x, y \in X)(\mu(x) \geq \kappa\min\{\mu(x * y), \mu(y)\})$.

Definition 4.7. A fuzzy subset μ of a BCI-algebra X is called a *fuzzy q-ideal* of X with degree (λ, κ) if it satisfies:

(1) $(\forall x \in X)(\mu(0) \geq \lambda\mu(x))$,
(2) $(\forall x, y, z \in X)(\mu(x * z) \geq \kappa\min\{\mu(x * (y * z)), \mu(y)\})$.

Note that if $\lambda \neq \kappa$, then a fuzzy q-ideal with degree (λ, κ) may not be a fuzzy q-ideal with degree (κ, λ), and vice versa.

Example 4.8. Let $X = \{0, a, b\}$ be a BCI-algebra([6]) in which the $*$-operation is given by the following table:

*	0	a	b
0	0	0	b
a	a	0	b
b	b	b	0

Define a fuzzy subset $\mu : X \to [0,1]$ by

$$\mu = \begin{pmatrix} 0 & a & b \\ 0.7 & 0.8 & 0.4 \end{pmatrix}$$

Then μ is a fuzzy q-ideal of X with degree $(\frac{5}{6}, \frac{3}{6})$ but it is not a fuzzy q-ideal of X since

$$\mu(0) = 0.7 \ngeq 0.8 = \mu(a).$$

Obviously, every fuzzy q-ideal is a fuzzy q-ideal with degree (λ, κ), but the converse may not be true. In fact, the fuzzy q-ideal μ with degree $(\frac{5}{6}, \frac{3}{6})$ in Example 4.8 is not a fuzzy q-ideal of X. Note that a fuzzy q-ideal with degree (λ, κ) is a fuzzy q-ideal if and only if $(\lambda, \kappa) = (1, 1)$.

PROPOSITION 4.9. *If μ is a fuzzy q-ideal of a BCI-algebra X with degree (λ, κ), then μ is a fuzzy ideal of X with degree (λ, κ).*

Proof. Put $z := 0$ in Definition 4.7(2). □

The converse of Proposition 4.9 is not true in general as seen the following example.

EXAMPLE 4.10. Let $X = \{0, a, 1, 2, 3\}$ be a BCI-algebra([4]) in which the $*$-operation is given by the following table:

$*$	0	a	1	2	3
0	0	0	3	2	1
a	a	0	3	2	1
1	1	1	0	3	2
2	2	2	1	0	3
3	3	3	2	1	0

Define a fuzzy subset $\mu : X \to [0,1]$ by

$$\mu = \begin{pmatrix} 0 & a & 1 & 2 & 3 \\ 0.8 & 0.6 & 0.5 & 0.5 & 0.5 \end{pmatrix}$$

It is routine to check that μ is a fuzzy ideal of X with degree $(\frac{4}{7}, \frac{4}{5})$. But it is not a fuzzy q-ideal of degree $(\frac{4}{7}, \frac{4}{5})$, since

$$\mu(3 * 1) = 0.5 \ngeq \frac{4}{5} \times 0.8 = \frac{4}{5} \min\{\mu(3 * (0 * 1)), \mu(0)\}.$$

PROPOSITION 4.11. *If μ is a fuzzy q-ideal of a BCI-algebra X with degree (λ, κ), then*

(1) $(\forall x, y \in X)(x \leq y \Rightarrow \mu(x) \geq \lambda\kappa\mu(y))$.

(2) $(\forall x, y \in X)(\mu(x * y) \geq \lambda\kappa\mu(x * (0 * y))$.
(3) $(\forall x, y, z \in X)(\mu((x * y) * z)) \geq \lambda^2\kappa^2\mu(x * (y * z))$.

Proof. (1) Let $x, y \in X$ be such that $x \leq y$. Then $x * y = 0$. Putting $z := 0$ in Definition 4.7(2) and using (b1), we have

$$\begin{aligned}\mu(x) = \mu(x * 0) \geq& \kappa\min\{\mu(x * (y * 0)), \mu(y)\}\\ =&\kappa\min\{\mu(x * y), \mu(y)\}\\ =&\kappa\min\{\mu(0), \mu(y)\}\\ \geq&\kappa\min\{\lambda\mu(y), \mu(y)\}\\ =&\lambda\kappa\mu(y).\end{aligned}$$

(2) Let $x, y \in X$. Putting $x := x$, $y := 0$ and $z := y$ in Definition 4.7(2), we have

$$\begin{aligned}\mu(x * y) \geq&\kappa\min\{\mu(x * (0 * y)), \mu(0)\}\\ \geq&\kappa\min\{\mu(x * (0 * y)), \lambda\mu(x * (0 * y))\}\\ =&\kappa\lambda\mu(x * (0 * y)).\end{aligned}$$

(3) Since

$$\begin{aligned}((x * y) * (0 * z)) * (x * (y * z)) =&((x * y) * (x * (y * z)) * (0 * z)\\ \leq&((y * z) * y) * (0 * z)\\ =&(0 * z) * (0 * z) = 0 \ \forall x, y, z \in X,\end{aligned}$$

we get $(x * y) * (0 * z) \leq x * (y * z)$. It follows from (2) and Proposition 4.11(1) that

$$\begin{aligned}\mu((x * y) * z) \geq&\kappa\lambda\mu((x * y) * (0 * z))\\ \geq&\kappa^2\lambda^2\mu(x * (y * z)).\end{aligned}$$

□

Corollary 4.12. *Let μ be a fuzzy q-ideal of a BCI-algebra X with degree (λ, κ). If $\lambda = \kappa$, then the following hold:*

(1) $(\forall x, y \in X)(x \leq y \Rightarrow \mu(x) \geq \lambda^2\mu(y))$.
(2) $(\forall x, y \in X)(\mu(x * y) \geq \lambda^2\mu(x * (0 * y))$.
(3) $(\forall x, y, z \in X)(\mu((x * y) * z) \geq \lambda^4\mu(x * (y * z))$.

Proposition 4.13. *Let μ be a fuzzy ideal of X with degree with (λ, κ). If $\mu(x) \leq \mu(x * y)$ for any $x, y \in X$, then μ is a fuzzy q-ideal of X with degree (λ, κ).*

Proof. For any $x, y, z \in X$, we have $\mu(x) \geq \kappa\min\{\mu(x*(y*z)), \mu(y*z)\}$. By assumption, we obtain

$$\begin{aligned}\mu(x*z) \geq & \mu(x) \\ & \geq \kappa\min\{\mu(x*(y*z)), \mu(y*z)\} \\ & \geq \kappa\min\{\mu(x*(y*z)), \mu(y)\}.\end{aligned}$$

Thus μ is a fuzzy q-ideal of X. □

Denote by $\mathcal{I}(X)$ and $\mathcal{I}_q(X)$ the set of all ideals and q-ideals of a BCI-algebra X, respectively. Note that a fuzzy subset μ of a BCI-algebra X is a fuzzy q-ideal of X if and only if

$$(\forall t \in [0, 1])(U(\mu; t) \in \mathcal{I}_q(X) \cup \{\emptyset\}).$$

But we know that for any fuzzy subset μ of a BCI-algebra X there exist $\lambda, \kappa \in (0, 1)$ and $t \in [0, 1]$ such that

(1) μ is a fuzzy q-ideal of X with degree (λ, κ),
(2) $U(\mu; t) \notin \mathcal{I}_q(X) \cup \{\emptyset\}$.

EXAMPLE 4.14. Consider a BCI-algebra $X = \{0, a, b, c\}$ as in Example 3.4. Define a fuzzy subset $\mu : X \to [0, 1]$ by

$$\mu = \begin{pmatrix} 0 & a & b & c \\ 0.7 & 0.6 & 0.5 & 0.5 \end{pmatrix}$$

Then μ is a fuzzy q-ideal of X with degree $(0.6, 0.7)$. If $t \in (0.6, 0.7]$, then $U(\mu; t) = \{0\}$ is not a q-ideal of X since $c*(0*a) = 0 \in \{0\}$, $0 \in \{0\}$ and $c*a = b \notin \{0\}$.

THEOREM 4.15. *Let μ be a fuzzy subset of a BCI-algebra X. For any $t \in [0, 1]$ with $t \leq \max\{\lambda, \kappa\}$, if $U(\mu; t)$ is an enlarged q-ideal of X related to $U(\mu; \frac{t}{\max\{\lambda,\kappa\}})$, then μ is a fuzzy q-ideal of X with degree (λ, κ).*

Proof. Assume that $\mu(0) < t \leq \lambda\mu(x)$ for some $x \in X$ and $t \in (0, \lambda]$. Then $\mu(x) \geq \frac{t}{\lambda} \geq \frac{t}{\max\{\lambda,\kappa\}}$ and so $x \in U(\mu; \frac{t}{\max\{\lambda,\kappa\}})$, i.e., $U(\mu; \frac{t}{\max\{\lambda,\kappa\}}) \neq \emptyset$. Since $U(\mu; t)$ is an enlarged q-ideal of X related to $U(\mu; \frac{t}{\max\{\lambda,\kappa\}})$, we have $0 \in U(\mu; t)$, i.e., $\mu(0) \geq t$. This is a contradiction, and thus $\mu(0) \geq \lambda\mu(x)$ for all $x \in X$.

Now suppose that there exist $a, b, c \in X$ such that $\mu(a*c) < \kappa\min\{\mu(a*(b*c)), \mu(b)\}$. If we take $t := \kappa\min\{\mu(a*(b*c)), \mu(b)\}$, then $t \in (0, \kappa] \subseteq (0, \max\{\lambda, \kappa\}]$. Hence $a*(b*c) \in U(\mu; \frac{t}{\kappa}) \subseteq U(\mu; \frac{t}{\max\{\lambda,\kappa\}})$

and $b \in U(\mu; \frac{t}{\kappa}) \subseteq U(\mu; \frac{t}{\max\{\lambda,\kappa\}})$. It follows from Definition 4.2(3) that $a * c \in U(\mu; t)$ so that $\mu(a * c) \geq t$, which is impossible. Therefore

$$\mu(x * z) \geq \kappa \min\{\mu(x * (y * z)), \mu(y)\}$$

for all $x, y, z \in X$. Thus μ is a fuzzy q-ideal of X with degree (λ, κ). □

COROLLARY 4.16. *Let μ be a fuzzy subset of a BCI-algebra X. For any $t \in [0, 1]$ with $t \leq \frac{k}{n}$, if $U(\mu; t)$ is an enlarged q-ideal of X related to $U(\mu; \frac{n}{k}t)$, then μ is a fuzzy q-ideal of X with degree $(\frac{k}{n}, \frac{k}{n})$.*

THEOREM 4.17. *Let $t \in [0, 1]$ be such that $U(\mu; t)(\neq \emptyset)$ is not necessary a q-ideal of a BCI-algebra X. If μ is a fuzzy q-ideal of X with degree (λ, κ), then $U(\mu; t\min\{\lambda, \kappa\})$ is an enlarged q-ideal of X related to $U(\mu; t)$.*

Proof. Since $t\min\{\lambda, \kappa\} \leq t$, we have $U(\mu; t) \subseteq U(\mu; t\min\{\lambda, \kappa\})$. Since $U(\mu; t) \neq \emptyset$, there exists $x \in U(\mu; t)$ and so $\mu(x) \geq t$. By Definition 4.7(1), we obtain $\mu(0) \geq \lambda\mu(x) \geq \lambda t \geq t\min\{\lambda, \kappa\}$. Therefore $0 \in U(\mu; t\min\{\lambda, \kappa\})$.

Let $x, y, z \in X$ be such that $x * (y * z) \in U(\mu; t)$ and $y \in U(\mu; t)$. Then $\mu(x * (y * z)) \geq t$ and $\mu(y) \geq t$. It follows from Definition 4.7(2) that

$$\begin{aligned}\mu(x * z) \geq &\kappa \min\{\mu(x * (y * z)), \mu(y)\} \\ \geq &\kappa t \geq t\min\{\lambda, \kappa\}.\end{aligned}$$

so that $x * z \in U(\mu; t\min\{\lambda, \kappa\})$. Thus $U(\mu; t\min\{\lambda, \kappa\})$ is an enlarged q-ideal of X related to $U(\mu; t)$. □

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Department of Mathematics Education
Dongguk University
Seoul 10-715, Republic of Korea
E-mail: tprud-tjd@hanmail.net

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Department of Mathematics Eduaction
Dongguk University
Seoul 100-715, Republic of Korea
E-mail: sunshine@dongguk.edu