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APPROXIMATELY CENTRALIZING DERIVATIONS OF NONCOMMUTATIVE BANACH ALGEBRA

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Abstract. In this paper, we consider the functional inequalities with approximately centralizing derivations on noncommutative Banach algebras, and investigate the problem that functions satisfying the functional inequalities mentioned above map into the radical.

1. Introduction and preliminaries

Throughout this paper, $\mathcal{X}$ will denote algebra over the real or complex field $\mathbb{F}$. A mapping $L : \mathcal{X} \to \mathcal{X}$ is called a *centralizing* if the functional equation $[L(x), x] \in Z(\mathcal{X})$ for all $x \in \mathcal{X}$, where, $[x, y] = xy - yx$ is the commutator of x and y, and $Z(\mathcal{X})$ is the center of $\mathcal{X}$. An additive mapping $d : \mathcal{X} \to \mathcal{X}$ is called a *ring derivation* if the functional equation $d(xy) = xd(y) + d(x)y$ is valid for all $x, y \in \mathcal{X}$. In addition, d is said to be a *linear derivation* if the functional equation $d(\lambda x) = \lambda d(x)$ holds for all $\lambda \in \mathbb{F}$ and all $x \in \mathcal{X}$.

Let us introduce the historical background of our investigation. The stability problem of functional equations has originally been formulated by Ulam [16] : *under what condition does there exists a homomorphism near an approximate homomorphism?* Hyers [8] answered the problem of Ulam under the assumption that the groups are Banach spaces. A generalized version of the theorem of Hyers for approximately additive mappings was given by Aoki [1] and for approximately linear mappings was presented by Rassias [12] by considering an unbounded Cauchy difference. The paper work of Rassias [12] has had a lot of influence in

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the development of what call the *generalized Hyers-Ulam stability* of functional equations.

Since then, more generalizations and applications of the generalized Hyers-Ulam stability to a number of functional equations and mappings have been investigated. In particular, the stability result concerning derivations between operator algebras was first obtained by Šemrl [13]. Badora [2] gave a generalization of the Bourgin's result [5] and he also dealt with the Hyers-Ulam stability and the Bourgin-type superstability of derivations in [3].

Singer and Wermer [14] obtained a fundamental result which started investigation into the ranges of linear derivations on Banach algebras. The result, which is called the Singer-Wermer theorem, states that every continuous linear derivation on a commutative Banach algebra maps into the radical. They also made a very insightful conjecture, namely that the assumption of continuity is unnecessary. This was known as the Singer-Wermer conjecture and was proved by Thomas [15]. The Singer-Wermer conjecture implies that every linear derivation on a commutative semisimple Banach algebra is identically zero which is the result of Johnson [9]. On the other hand, Hatori and Wada [7] showed that a zero operator is the only ring derivation on a commutative semisimple Banach algebra with the maximal ideal space without isolated points. Note that this differs from the above result of Johnson. Based on these facts and a private communication with Watanabe [11], Miura et al. proved the generalized Hyers-Ulam stability and Bourgin-type superstability of ring derivations on Banach algebras in [11].

The main purpose of this paper is to study the superstability for functional inequalities with centralizing derivations and investigate the problem of functional inequalities which derivations map into the radical of noncommutative Banach algebras.

2. Main results

We first demonstrate the definition which is used in this section.

DEFINITION 2.1. Let $\mathcal{X}$ be an algebra. A linear mapping $f : \mathcal{X} \to \mathcal{X}$ is said to be an *approximately centralizing linear derivation* if f is a linear derivation such that

$$\|[f(x), x]y - y[f(x), x]\| \leq \delta$$

is fulfilled for all $x, y \in \mathcal{X}$.

In view of the Thomas' result [15], derivations on Banach algebras now belong to the noncommutative setting. Among the various noncommutative versions of the Singer-Wermer theorem, Breŝar [6] proved that *every centralizing linear derivation on a semiprime Banach algebra maps into the intersection of the center and the radical* and Mathieu and Murphy [10] verify that *every continuous centralizing linear derivation on a Banach algebra maps into the radical.*

Theorem 2.2. *Let $\mathcal{X}$ be a noncommutative Banach algebra. Assume that s, t are fixed positive numbers, and $r = \max\{s, t\}$ and $u = \min\{s, t\}$. Suppose that $f : \mathcal{X} \to \mathcal{X}$ is a continuous mapping subjected to the inequality*

$$\begin{aligned} &\|\alpha s f(x) + \alpha t f(y) + r f(\alpha z) - r u f(v) - r f(u) v\| \\ &\quad \leq \Big\| r f\Big(\frac{sx + ty}{r} + z - uv\Big)\Big\| \end{aligned} \tag{2.1}$$

for all $x, y, z, u, v \in \mathcal{X}$, where $\alpha = 1, i$, and

$$\|[f(x), x]y - y[f(x), x]\| \leq \delta \tag{2.2}$$

for all $x, y \in \mathcal{X}$. Then f maps $\mathcal{X}$ into the radical $rad(\mathcal{X})$.

Proof. By letting $\alpha = 1$ and $x = y = z = u = v = 0$ in (2.1), we get $f(0) = 0$. And by putting $u = 0$ in (2.1), we have

$$\|\alpha s f(x) + \alpha t f(y) + r f(\alpha z)\| \leq \Big\| r f\Big(\frac{sx + ty}{r} + z\Big)\Big\| \tag{2.3}$$

for all $x, y, z \in \mathcal{A}$. Without loss of generality, we assume that $r = t$. Then (2.1) can be rewritten the following

$$\|\alpha u f(x) + \alpha r f(y) + r f(\alpha z)\| \leq \Big\| r f\Big(\frac{ux}{r} + y + z\Big)\Big\| \tag{2.4}$$

for all $x, y, z \in \mathcal{A}$. Also, by setting $\alpha = 1$, $x = 0$, $y = x$ and $z = -x$ in (2.4), we obtain $f(-x) = -f(x)$ for all $x \in \mathcal{A}$. Letting $\alpha = 1$, $x = \frac{r}{u}(-x - y)$, $y = x$ and $z = y$ in (2.4), we arrive at

$$r f(x) + r f(y) = u f\Big(\frac{r}{u}(x + y)\Big) \tag{2.5}$$

for all $x, y \in \mathcal{A}$. By putting $y = 0$ in (2.5), we get

$$f\Big(\frac{r}{u} x\Big) = \frac{r}{u} f(x) \tag{2.6}$$

for all $x \in \mathcal{A}$. Comparing (2.5) and (2.6), we see that f is additive.

Now, since f is continuous, we feel that $f(tx)$ is continuous in $t \in \mathbb{R}$ for each fixed $x \in \mathcal{X}$. The mapping f is $\mathbb{R}$-linear [12]. By letting $\alpha =$

i, $x = 0$, $y = x$ and $z = -x$ in (2.4), we obtain $f(ix) = if(x)$ for all $x \in \mathcal{X}$. Therefore, for all $\mu = s + it \in \mathbb{C}$ and all $x \in \mathcal{X}$,

$$f(\mu x) = sf(x) + itf(x) = \mu f(x).$$

Thus, we conclude that f is $\mathbb{C}$-linear.

By letting $\alpha = 1$ and $x = y = 0$, $z = xy$, $u = x$, $v = y$ in (2.1), we arrive at $f(xy) = xf(y) + f(x)y$.

Therefore, f is linear derivation satisfying (2.2), that is, f is approximately centralizing derivation

On the other hand, it follows by (2.6) that

$$f(x) = \frac{u}{r} f\Big(\frac{r}{u}x\Big) = \Big(\frac{u}{r}\Big)^2 f\Big(\Big(\frac{r}{u}\Big)^2 x\Big) = \cdots = \Big(\frac{u}{r}\Big)^n f\Big(\Big(\frac{r}{u}\Big)^n x\Big) \tag{2.7}$$

for all positive integer n and all $x \in \mathcal{X}$. So we now define

$$f(x) := \lim_{n\to\infty} \Big(\frac{u}{r}\Big)^n f\Big(\Big(\frac{r}{u}\Big)^n x\Big) \tag{2.8}$$

for all $x \in \mathcal{A}$. In view of (2.2) and (2.8), we have

$$\begin{aligned}
&\|[f(x), x]y - y[f(x), x]\| \\
&= \lim_{n\to\infty} \Big(\frac{u}{r}\Big)^{2n} \Big\| \Big(\frac{r}{u}\Big)^n \Big[f\Big(\Big(\frac{r}{u}\Big)^n x\Big), x\Big]y - \Big(\frac{r}{u}\Big)^n y\Big[f\Big(\Big(\frac{r}{u}\Big)^n x\Big), x\Big]\Big\| \\
&\le \lim_{n\to\infty} \Big(\frac{u}{r}\Big)^{2n} \delta = 0,
\end{aligned}$$

which means that f is a centralizing mapping.

With the help of Mathieu and Murphy's result, we have the desired assertion. □

THEOREM 2.3. *Let $\mathcal{X}$ be a noncommutative semiprime Banach algebra. Assume that s, t are fixed positive numbers, and $r = \max\{s, t\}$ and $u = \min\{s, t\}$. Suppose that $f : \mathcal{X} \to \mathcal{X}$ is a mapping subjected to the inequality*

$$\begin{aligned}
&\|\alpha sf(x) + \alpha tf(y) + rf(\alpha z) - ruf(v) - rf(u)v\| \\
&\quad \le \Big\| rf\Big(\frac{sx + ty}{r} + z - uv\Big)\Big\|
\end{aligned} \tag{2.9}$$

for all $x, y, z, u, v \in \mathcal{X}$ and all $\alpha \in \mathbb{U} := \{z \in \mathbb{C} : |z| = 1\}$, and the inequality (2.2). Then f maps $\mathcal{X}$ into the intersection of the center $Z(\mathcal{X})$ and the radical $rad(\mathcal{X})$.

Proof. As in the proof of Theorem 2.2, we have $f(0) = 0$ and

$$\|\alpha sf(x) + \alpha tf(y) + rf(\alpha z)\| \le \Big\| rf\Big(\frac{sx + ty}{r} + z\Big)\Big\|$$

for all $x, y, z \in \mathcal{X}$ and all $\alpha \in \mathbb{U}$. Without loss of generality, we assume that $r = t$. The above functional inequality can be written as follows.

$$\|\alpha s f(x) + \alpha r f(y) + r f(\alpha z)\| \le \Big\| r f\Big(\frac{ux}{r} + y + z\Big)\Big\| \tag{2.10}$$

for all $x, y, z \in \mathcal{X}$ and all $\alpha \in \mathbb{U}$. Employing the same argument as in the proof of Theorem 2.2, we find that f is a centralizing ring derivation.

Now we need to show that f is linear: By letting $x = 0$, $y = x$ and $z = -x$ in (2.10), we obtain $f(\alpha x) = \alpha f(x)$. Clearly, $f(0x) = 0 = 0f(x)$. Let us assume that λ is a nonzero complex number and that M is a positive integer greater than $|\lambda|$. Then, by applying a geometric argument, there exist $\lambda_1, \lambda_2 \in \mathbb{U}$ such that $2(\lambda/M) = \lambda_1 + \lambda_2$. In particular, we obtain $f(x/2) = (1/2)f(x)$ for all $x \in \mathcal{A}$. Thus we have that $f(\lambda x) = \lambda f(x)$ for all $x \in \mathcal{A}$, so that f is $\mathbb{C}$-linear.

According to Brešar's result, we arrive at the conclusion of Theorem. □

THEOREM 2.4. *Let $\mathcal{X}$ be a noncommutative Banach algebra with unit. Assume that s, t are fixed distinct positive numbers, and $r = \max\{s, t\}$ and $u = \min\{s, t\}$. Suppose that $f : \mathcal{X} \to \mathcal{X}$ is a continuous mapping subjected to the inequality*

$$\begin{aligned}\|\alpha s f(x) + \alpha t f(y) + r f(\alpha z) - r u f(v) - r f(u) v\| \\ \le \Big\| r f\Big(\frac{sx + ty}{r} + z - uv\Big)\Big\| + \varepsilon\end{aligned} \tag{2.11}$$

for all $x, y, z, u, v \in \mathcal{X}$, where $\alpha = 1, i$, and the inequality (2.2). Then f maps $\mathcal{X}$ into the radical $rad(\mathcal{X})$.

Proof. By letting $\alpha = 1$ and $x = y = z = u = v = 0$ in (2.11), we get $\|f(0)\| \le \frac{\varepsilon}{u+r}$. Without loss of generality, we assume that $r = t$. Then, (2.11) can be written as the following:

$$\begin{aligned}\|\alpha u f(x) + \alpha r f(y) + r f(\alpha z) - r u f(v) - r f(u) v\| \\ \le \Big\| r f\Big(\frac{ux}{r} + y + z - uv\Big)\Big\| + \varepsilon\end{aligned} \tag{2.12}$$

for all $x, y, z, u, v \in \mathcal{X}$ with $\alpha = 1, i$. We also have by letting $\alpha = 1$, $x = \frac{r}{u}x$, $y = -x$, $z = 0$ and $u = v = 0$ in (2.12) that

$$\Big\| \frac{u}{r} f\Big(\frac{r}{u}x\Big) + f(-x)\Big\| \le \Big(\frac{1}{r} + \frac{2}{u+r}\Big)\varepsilon \tag{2.13}$$

for all $x \in \mathcal{X}$. Next, by letting $\alpha = 1$, $x = 0$, $y = x$, $z = -x$ and $u = v = 0$ in (2.12), we obtain

$$\|f(x) + f(-x)\| \le \frac{2\varepsilon}{r} \tag{2.14}$$

for all $x \in \mathcal{X}$. Therefore, by (2.13) and (2.14), we see that

$$\begin{aligned}
&\Big\|\Big(\frac{u}{r}\Big)^{l} f\Big(\Big(\frac{r}{u}\Big)^{l} x\Big) - \Big(\frac{u}{r}\Big)^{m} f\Big(\Big(\frac{r}{u}\Big)^{m} x\Big)\Big\| \\
&\le \sum_{j=l}^{m-1} \Big\|\Big(\frac{u}{r}\Big)^{j} f\Big(\Big(\frac{r}{u}\Big)^{j} x\Big) - \Big(\frac{u}{r}\Big)^{j+1} f\Big(\Big(\frac{r}{u}\Big)^{j+1} x\Big)\Big\| \\
&\le \sum_{j=l}^{m-1} \Big[\Big\|\Big(\frac{u}{r}\Big)^{j} f\Big(\Big(\frac{r}{u}\Big)^{j} x\Big) + \Big(\frac{u}{r}\Big)^{j+1} f\Big(-\Big(\frac{r}{u}\Big)^{j+1} x\Big)\Big\| \\
&\quad + \Big\|\Big(\frac{u}{r}\Big)^{j+1} f\Big(-\Big(\frac{r}{u}\Big)^{j+1} x\Big) + \Big(\frac{u}{r}\Big)^{j+1} f\Big(\Big(\frac{r}{u}\Big)^{j+1} x\Big)\Big\|\Big] \\
&\le \sum_{j=l}^{m-1} \Big(\frac{u}{r}\Big)^{j} \Big[\frac{2u}{r^2} + \frac{1}{r} + \frac{2}{u+r}\Big]\varepsilon
\end{aligned}$$

for all nonnegative integers m, l with $m > l$ and all $x \in \mathcal{X}$. It follows that the sequence $\{(\frac{u}{r})^n f((\frac{r}{u})^n x)\}$ is a Cauchy and so it is convergent. So one can define a mapping $L : \mathcal{X} \to \mathcal{X}$ by $L(x) := \lim_{n\to\infty}(\frac{u}{r})^n f((\frac{r}{u})^n x)$ for all $x \in \mathcal{X}$. By letting $l = 0$ and taking the limit $m \to \infty$, we obtain

$$\|f(x) - L(x)\| \le \Big[\frac{2u}{r^2} + \frac{1}{r} + \frac{2}{u+r}\Big]\frac{r\varepsilon}{r-u} \tag{2.15}$$

for all $x \in \mathcal{X}$.

Now, we claim that the mapping L is additive. By (2.14), one notes

$$\begin{aligned}
\|L(x) + L(-x)\| &= \lim_{n\to\infty} \Big(\frac{u}{r}\Big)^{n} \Big\|f\Big(\Big(\frac{r}{u}\Big)^{n} x\Big) + f\Big(-\Big(\frac{r}{u}\Big)^{n} x\Big)\Big\| \\
&\le \lim_{n\to\infty} \Big(\frac{u}{r}\Big)^{n} \cdot \frac{2\varepsilon}{r} = 0.
\end{aligned}$$

So we have $L(-x) = -L(x)$. By (2.12), we arrive at

$$\begin{aligned}
&\|L(x) + L(y) - L(x+y)\| \\
&= \lim_{n\to\infty} \Big(\frac{u}{r}\Big)^{n} \Big\|f\Big(\Big(\frac{r}{u}\Big)^{n} x\Big) + f\Big(\Big(\frac{r}{u}\Big)^{n} y\Big) + \frac{u}{r} f\Big(\Big(\frac{r}{u}\Big)^{n+1}(-x-y)\Big)\Big\| \\
&\le \lim_{n\to\infty} \Big(\frac{u}{r}\Big)^{n} \Big[\frac{1}{u+r} + \frac{1}{r}\Big]\varepsilon = 0
\end{aligned}$$

for all $x, y \in \mathcal{X}$. So L is additive mapping.

Now, to show uniqueness of the mapping L, let us assume that $T : \mathcal{X} \to \mathcal{X}$ is another additive mapping satisfying (2.15). Then we have

$$\begin{aligned}\|L(x) - T(x)\| &= \lim_{n\to\infty} \Big(\frac{u}{r}\Big)^n \Big\| L\Big(\Big(\frac{r}{u}\Big)^n x\Big) - T\Big(\Big(\frac{r}{u}\Big)^n x\Big)\Big\| \\ &\leq \lim_{n\to\infty} \Big(\frac{u}{r}\Big)^n \Big[\Big\| L\Big(\Big(\frac{r}{u}\Big)^n x\Big) - f\Big(\Big(\frac{r}{u}\Big)^n x\Big)\Big\| \\ &\quad + \Big\| f\Big(\Big(\frac{r}{u}\Big)^n x\Big) - L\Big(\Big(\frac{r}{u}\Big)^n x\Big)\Big\| \Big] \\ &\leq \lim_{n\to\infty} 2\Big(\frac{u}{r}\Big)^n \Big[\frac{2u}{r^2} + \frac{1}{r} + \frac{2}{u+r}\Big]\frac{r\varepsilon}{r-u} = 0\end{aligned}$$

for all $x \in \mathcal{X}$. Therefore we conclude that there exists a unique additive mapping $L : \mathcal{X} \to \mathcal{X}$ satisfying (2.15).

By letting $\alpha = 1$, $x = y = 0$, $z = xy$, $u = x, v = y$ in (2.12), we get

$$\|f(xy) - xf(y) - f(x)y\| \leq \Big[\frac{2r+u}{r+u} + 1\Big]\frac{\varepsilon}{r},$$

which implies that

$$\begin{aligned}&\lim_{n\to\infty} \Big(\frac{u}{r}\Big)^n \Big\| f\Big(\Big(\frac{r}{u}\Big)^n xy\Big) - \Big(\frac{r}{u}\Big)^n xf(y) - f\Big(\Big(\frac{r}{u}\Big)^n x\Big)y\Big\| \\ &\leq \lim_{n\to\infty} \Big(\frac{r}{u}\Big)^n \Big[\frac{2r+u}{r+u} + 1\Big]\frac{\varepsilon}{r} = 0.\end{aligned}$$

That is,

$$L(xy) = xf(y) + L(x)y \tag{2.16}$$

for all $x, y \in \mathcal{X}$. It follows from (2.16) that

$$\begin{aligned}\Big(\frac{r}{u}\Big)^n xf(y) + \Big(\frac{r}{u}\Big)^n L(x)y &= L\Big(\Big(\frac{r}{u}\Big)^n x \cdot y\Big) \\ &= L\Big(x \cdot \Big(\frac{r}{u}\Big)^n y\Big) = xf\Big(\Big(\frac{r}{u}\Big)^n y\Big) + \Big(\frac{r}{u}\Big)^n L(x)y.\end{aligned}$$

Hence we have

$$xf(y) = \lim_{n\to\infty} x\Big(\frac{u}{r}\Big)^n f\Big(\Big(\frac{r}{u}\Big)^n y\Big) = xL(y)$$

Since $\mathcal{X}$ contains the unit element, we see that $L = f$. Therefore, f is a ring derivation.

Now, since f is continuous, we feel that $f(tx)$ is continuous in $t \in \mathbb{R}$ for each fixed $x \in \mathcal{X}$. The mapping f is $\mathbb{R}$-linear [12]. Again, by letting $\alpha = i$, $x = 0$, $y = x$, $z = -x$ and $u = v = 0$ in (2.12), we obtain

$$\|if(x) + f(-ix)\| \leq \frac{2\varepsilon}{r} \tag{2.17}$$

for all $x \in \mathcal{X}$. Due to (2.17), we have

$$\lim_{n\to\infty}\Big(\frac{u}{r}\Big)^n \Big\| if\Big(\Big(\frac{r}{u}\Big)^n x\Big) + f\Big(-i\Big(\frac{r}{u}\Big)^n x\Big)\Big\| \le \lim_{n\to\infty}\Big(\frac{u}{r}\Big)^n \frac{2\varepsilon}{r} = 0,$$

which means that $f(ix) = if(x)$ for all $x \in \mathcal{X}$. As in the proof of Theorem 2.2, the mapping f is $\mathbb{C}$-linear.

Thus, f is linear derivation satisfying (2.2), that is, f is approximately centralizing derivation and employing the same argument as the proof Theorem 2.2, we know that f is centralizing mapping.

The Mathieu and Murphy's result guarantees the claim of the theorem. □

THEOREM 2.5. *Let $\mathcal{X}$ be a noncommutative semiprime Banach algebra with unit. Assume that s, t are fixed positive numbers, and $r = \max\{s,t\}$ and $u = \min\{s,t\}$. Suppose that $f : \mathcal{X} \to \mathcal{X}$ is a mapping subjected to the inequality (2.11)*

$$\begin{aligned}&\|\alpha s f(x) + \alpha t f(y) + r f(\alpha z) - r u f(v) - r f(u) v\| \\ &\quad \le \Big\| r f\Big(\frac{sx+ty}{r} + z - uv\Big)\Big\| + \varepsilon\end{aligned}$$

for all $x, y, z, u, v \in \mathcal{X}$ and all $\alpha \in \mathbb{U}$, and the inequality (2.2). Then f maps $\mathcal{X}$ into the intersection of the center $Z(\mathcal{X})$ and the radical $rad(\mathcal{X})$.

Proof. As we did in the proof of Theorem 2.3, we find that f is a centralizing ring derivation.

Again, by letting $x = 0$, $y = x$, $z = -x$ and $u = v = 0$ in (2.12), we obtain

$$\|\alpha f(x) + f(-\alpha x)\| \le \frac{2\varepsilon}{r} \tag{2.18}$$

for all $x \in \mathcal{X}$. Due to (2.18), we have

$$\lim_{n\to\infty}\Big(\frac{u}{r}\Big)^n \Big\| \alpha f\Big(\Big(\frac{r}{u}\Big)^n x\Big) + f\Big(-\alpha\Big(\frac{r}{u}\Big)^n x\Big)\Big\| \le \lim_{n\to\infty}\Big(\frac{u}{r}\Big)^n \frac{2\varepsilon}{r} = 0,$$

which means that $f(\alpha x) = \alpha f(x)$ for all $x \in \mathcal{X}$. As in the proof of Theorem 2.3, the mapping f is $\mathbb{C}$-linear.

From the Brešar's result, we come to the concluding remark of the theorem. □

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