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FUZZY WEAKLY (r,s)-SEMICONTINUOUS MAPPINGS

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Abstract. In this paper, we introduce the concept of fuzzy weakly (r,s)-semicontinuous mappings on intuitionistic fuzzy topological spaces in Šostak's sense. The relations among various kinds of fuzzy mappings on intuitionistic fuzzy topological spaces in Šostak's sense are displayed. The characterization for the fuzzy weakly (r,s)-semicontinuous mapping is obtained. Also, we introduce the notion of fuzzy weakly (r,s)-semicontinuous mappings at a given intuitionistic fuzzy point. The relation between fuzzy weakly (r,s)-semicontinuous mappings and fuzzy weakly (r,s)-semicontinuous mappings at an intuitionistic fuzzy point is discussed.

1. Introduction

The concept of fuzzy set was introduced by Zadeh [13]. Chang [3] defined fuzzy topological spaces. These spaces and its generalizations are later studied by several authors, one of which, developed by Šostak [12], used the idea of degree of openness. This type of generalization of fuzzy topological spaces was later rephrased by Chattopadhyay and his colleagues [4], and by Ramadan [11].

As a generalization of fuzzy sets, the concept of intuitionistic fuzzy sets was introduced by Atanassov [1]. Recently, Çoker and his colleagues [5, 7] introduced intuitionistic fuzzy topological spaces using intuitionistic fuzzy sets. Using the idea of degree of openness and degree of nonopenness, Çoker and Demirci [6] defined intuitionistic fuzzy topological spaces in Šostak's sense as a generalization of smooth topological spaces and intuitionistic fuzzy topological spaces. S. Z. Bai [2] introduced the concept of fuzzy weakly semicontinuous mappings on Chang's fuzzy topological spaces.

In this paper, we introduce the concept of fuzzy weakly (r,s)-semicontinuous mappings on intuitionistic fuzzy topological spaces in Šostak's sense. The relations among various kinds of fuzzy mappings on intuitionistic fuzzy topological

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spaces in Šostak's sense are displayed. The characterization for the fuzzy weakly (r, s)-semicontinuous mapping is obtained. Also, we introduce the notion of fuzzy weakly (r, s)-semicontinuous mappings at a given intuitionistic fuzzy point. The relation between fuzzy weakly (r, s)-semicontinuous mappings and fuzzy weakly (r, s)-semicontinuous mappings at an intuitionistic fuzzy point is discussed.

2. Preliminaries

We will denote the unit interval $[0, 1]$ of the real line by I. A member μ of I^X is called a *fuzzy set* in X. For any $\mu \in I^X$, μ^c denotes the complement $1 - \mu$. By $\tilde{0}$ and $\tilde{1}$ we denote constant maps on X with value 0 and 1, respectively. All other notations are standard notations of fuzzy set theory.

Let X be a nonempty set. An *intuitionistic fuzzy set* A is an ordered pair

$$A = (\mu_A, \gamma_A)$$

where the functions $\mu_A : X \to I$ and $\gamma_A : X \to I$ denote the degree of membership and the degree of nonmembership, respectively and $\mu_A + \gamma_A \leq 1$. Obviously every fuzzy set μ in X is an intuitionistic fuzzy set of the form $(\mu, \tilde{1} - \mu)$.

Definition 2.1. ([1]) Let $A = (\mu_A, \gamma_A)$ and $B = (\mu_B, \gamma_B)$ be intuitionistic fuzzy sets in X. Then

(1) $A \subseteq B$ iff $\mu_A \leq \mu_B$ and $\gamma_A \geq \gamma_B$.
(2) $A = B$ iff $A \subseteq B$ and $B \subseteq A$.
(3) $A^c = (\gamma_A, \mu_A)$.
(4) $A \cap B = (\mu_A \wedge \mu_B, \gamma_A \vee \gamma_B)$.
(5) $A \cup B = (\mu_A \vee \mu_B, \gamma_A \wedge \gamma_B)$.
(6) $\underline{0} = (\tilde{0}, \tilde{1})$ and $\underline{1} = (\tilde{1}, \tilde{0})$.

Let f be a map from a set X to a set Y. Let $A = (\mu_A, \gamma_A)$ be an intuitionistic fuzzy set in X and $B = (\mu_B, \gamma_B)$ an intuitionistic fuzzy set in Y. Then

(1) The image of A under f, denoted by $f(A)$, is an intuitionistic fuzzy set in Y defined by
$$f(A) = (f(\mu_A), \tilde{1} - f(\tilde{1} - \gamma_A)).$$
(2) The inverse image of B under f, denoted by $f^{-1}(B)$, is an intuitionistic fuzzy set in X defined by
$$f^{-1}(B) = (f^{-1}(\mu_B), f^{-1}(\gamma_B)).$$

A *smooth fuzzy topology* on X is a map $T : I^X \to I$ which satisfies the following properties:

(1) $T(\tilde{0}) = T(\tilde{1}) = 1$.

(2) $T(\mu_1 \wedge \mu_2) \geq T(\mu_1) \wedge T(\mu_2)$.
(3) $T(\bigvee \mu_i) \geq \bigwedge T(\mu_i)$.
The pair (X, T) is called a *smooth fuzzy topological space.*

An *intuitionistic fuzzy topology* on X is a family T of intuitionistic fuzzy sets in X which satisfies the following properties:
(1) $\underline{0},\ \underline{1} \in T$.
(2) If $A_1, A_2 \in T$, then $A_1 \cap A_2 \in T$.
(3) If $A_i \in T$ for each i, then $\bigcup A_i \in T$.
The pair (X, T) is called an *intuitionistic fuzzy topological space.*

Let $I(X)$ be a family of all intuitionistic fuzzy sets in X and let $I \otimes I$ be the set of the pair (r, s) such that $r, s \in I$ and $r + s \leq 1$.

Definition 2.2. ([6]) Let X be a nonempty set. An *intuitionistic fuzzy topology in Šostak's sense*(SoIFT for short) $\mathcal{T} = (\mathcal{T}_1, \mathcal{T}_2)$ on X is a map $\mathcal{T} : I(X) \to I \otimes I$ which satisfies the following properties:
(1) $\mathcal{T}_1(\underline{0}) = \mathcal{T}_1(\underline{1}) = 1$ and $\mathcal{T}_2(\underline{0}) = \mathcal{T}_2(\underline{1}) = 0$.
(2) $\mathcal{T}_1(A \cap B) \geq \mathcal{T}_1(A) \wedge \mathcal{T}_1(B)$ and $\mathcal{T}_2(A \cap B) \leq \mathcal{T}_2(A) \vee \mathcal{T}_2(B)$.
(3) $\mathcal{T}_1(\bigcup A_i) \geq \bigwedge \mathcal{T}_1(A_i)$ and $\mathcal{T}_2(\bigcup A_i) \leq \bigvee \mathcal{T}_2(A_i)$.
The $(X, \mathcal{T}) = (X, \mathcal{T}_1, \mathcal{T}_2)$ is said to be an *intuitionistic fuzzy topological space in Šostak's sense*(SoIFTS for short). Also, we call $\mathcal{T}_1(A)$ a *gradation of openness* of A and $\mathcal{T}_2(A)$ a *gradation of nonopenness* of A.

Definition 2.3. ([8]) Let A be an intuitionistic fuzzy set in SoIFTS $(X, \mathcal{T}_1, \mathcal{T}_2)$ and $(r, s) \in I \otimes I$. Then A is said to be
(1) *fuzzy (r, s)-open* if $\mathcal{T}_1(A) \geq r$ and $\mathcal{T}_2(A) \leq s$,
(2) *fuzzy (r, s)-closed* if $\mathcal{T}_1(A^c) \geq r$ and $\mathcal{T}_2(A^c) \leq s$.

Definition 2.4. ([8]) Let $(X, \mathcal{T}_1, \mathcal{T}_2)$ be a SoIFTS. For each $(r, s) \in I \otimes I$ and for each $A \in I(X)$, the *fuzzy (r, s)-interior* is defined by

$$\operatorname{int}(A, r, s) = \bigcup \{B \in I(X) \mid B \subseteq A,\ B \text{ is fuzzy } (r, s)\text{-open}\}$$

and the *fuzzy (r, s)-closure* is defined by

$$\operatorname{cl}(A, r, s) = \bigcap \{B \in I(X) \mid A \subseteq B,\ B \text{ is fuzzy } (r, s)\text{-closed}\}.$$

Let $(X, \mathcal{T}_1, \mathcal{T}_2)$ be an intuitionistic fuzzy topological space in Šostak's sense. Then it is easy to see that for each $(r, s) \in I \otimes I$, the family $\mathcal{T}_{(r,s)}$ defined by

$$\mathcal{T}_{(r,s)} = \{A \in I(X) \mid \mathcal{T}_1(A) \geq r \text{ and } \mathcal{T}_2(A) \leq s\}$$

is an intuitionistic fuzzy topology on X.

Let (X, T) be an intuitionistic fuzzy topological space and $(r, s) \in I \otimes I$. Then the map $T^{(r,s)} : I(X) \to I \otimes I$ defined by

$$T^{(r,s)}(A) = \begin{cases} (1,0) & \text{if } \ \mu = \underline{0}, \underline{1}, \\ (r,s) & \text{if } \ A \in T - \{\underline{0}, \underline{1}\}, \\ (0,1) & \text{otherwise} \end{cases}$$

becomes an intuitionistic fuzzy topology in Šostak's sense on X.

Let $\alpha, \beta \in [0,1]$ with $\alpha + \beta \leq 1$. An intuitionistic fuzzy point $x_{(\alpha,\beta)}$ in X is an intuitionistic fuzzy set in X defined by

$$x_{(\alpha,\beta)}(y) = \begin{cases} (\alpha, \beta) & \text{if } \ y = x, \\ (0,1) & \text{if } \ y \neq x. \end{cases}$$

In this case, x is called the *support* of $x_{(\alpha,\beta)}$, α the *value* of $x_{(\alpha,\beta)}$, and β the *nonvalue* of $x_{(\alpha,\beta)}$. An intuitionistic fuzzy point $x_{(\alpha,\beta)}$ is said to *belong* to an intuitionistic fuzzy set $A = (\mu_A, \gamma_A)$ in X, denoted by $x_{(\alpha,\beta)} \in A$, if $\mu_A(x) \geq \alpha$ and $\gamma_A(x) \leq \beta$. An intuitionistic fuzzy set A in X is the union of all intuitionistic fuzzy points which belong to A.

Definition 2.5. ([8, 9]) Let A be an intuitionistic fuzzy set in a SoIFTS $(X, \mathcal{T}_1, \mathcal{T}_2)$ and $(r, s) \in I \otimes I$. Then A is said to be

(1) *fuzzy* (r, s)*-semiopen* if there is a fuzzy (r, s)-open set B in X such that $B \subseteq A \subseteq \text{cl}(B, r, s)$,
(2) *fuzzy* (r, s)*-semiclosed* if there is a fuzzy (r, s)-closed set B in X such that $\text{int}(B, r, s) \subseteq A \subseteq B$,
(3) *fuzzy* (r, s)*-regular open* if $\text{int}(\text{cl}(A, r, s), r, s) = A$,
(4) *fuzzy* (r, s)*-regular closed* if $\text{cl}(\text{int}(A, r, s), r, s) = A$.

Theorem 2.6. ([8]) Let A be an intuitionistic fuzzy set in a SoIFTS $(X, \mathcal{T}_1, \mathcal{T}_2)$ and $(r, s) \in I \otimes I$. Then the following statements are equivalent:

(1) A is a fuzzy (r, s)-semiopen set.
(2) A^c is a fuzzy (r, s)-semiclosed set.
(3) $\text{cl}(\text{int}(A, r, s), r, s) \supseteq A$.
(4) $\text{int}(\text{cl}(A^c, r, s), r, s) \subseteq A^c$.

Definition 2.7. ([8]) Let $(X, \mathcal{T}_1, \mathcal{T}_2)$ be a SoIFTS. For each $(r, s) \in I \otimes I$ and for each $A \in I(X)$, the *fuzzy* (r, s)*-semiinterior* is defined by

$$\text{sint}(A, r, s) = \bigcup \{B \in I(X) \mid B \subseteq A, \ B \text{ is fuzzy } (r, s)\text{-semiopen}\}$$

and the *fuzzy* (r, s)*-semiclosure* is defined by

$$\text{scl}(A, r, s) = \bigcap \{B \in I(X) \mid A \subseteq B, \ B \text{ is fuzzy } (r, s)\text{-semiclosed}\}.$$

Obviously, $\mathrm{scl}(A,r,s)$ is the smallest fuzzy (r,s)-semiclosed set which contains A and $\mathrm{sint}(A,r,s)$ is the greatest fuzzy (r,s)-semiopen set which is contained in A. Also, $\mathrm{scl}(A,r,s)=A$ for any fuzzy (r,s)-semiclosed set A and $\mathrm{sint}(A,r,s)=A$ for any fuzzy (r,s)-semiopen set A. Moreover, we have

$$\mathrm{int}(A,r,s)\subseteq \mathrm{sint}(A,r,s)\subseteq A\subseteq \mathrm{scl}(A,r,s)\subseteq \mathrm{cl}(A,r,s).$$

Also, we have the following results:

(1) $\mathrm{scl}(\underline{0},r,s)=\underline{0}$, $\mathrm{scl}(\underline{1},r,s)=\underline{1}$.
(2) $\mathrm{scl}(A,r,s)\supseteq A$.
(3) $\mathrm{scl}(A\cup B,r,s)\supseteq \mathrm{scl}(A,r,s)\cup \mathrm{scl}(B,r,s)$.
(4) $\mathrm{scl}(\mathrm{scl}(A,r,s),r,s)=\mathrm{scl}(A,r,s)$.
(5) $\mathrm{sint}(\underline{0},r,s)=\underline{0}$, $\mathrm{sint}(\underline{1},r,s)=\underline{1}$.
(6) $\mathrm{sint}(A,r,s)\subseteq A$.
(7) $\mathrm{sint}(A\cap B,r,s)\subseteq \mathrm{sint}(A,r,s)\cap \mathrm{sint}(B,r,s)$.
(8) $\mathrm{sint}(\mathrm{sint}(A,r,s),r,s)=\mathrm{sint}(A,r,s)$.

Definition 2.8. ([8]) For an intuitionistic fuzzy set A in a SoIFTS $(X,\mathcal{T}_1,\mathcal{T}_2)$ and $(r,s)\in I\otimes I$, we have:

(1) $\mathrm{scl}(A,r,s)^c=\mathrm{sint}(A^c,r,s)$.
(2) $\mathrm{sint}(A,r,s)^c=\mathrm{scl}(A^c,r,s)$.

Definition 2.9. ([9, 10]) Let $f:(X,\mathcal{T}_1,\mathcal{T}_2)\to(Y,\mathcal{U}_1,\mathcal{U}_2)$ be a mapping from a SoIFTS X to a SoIFTS Y and $(r,s)\in I\otimes I$. Then f is called

(1) a *fuzzy (r,s)-continuous* mapping if $f^{-1}(B)$ is a fuzzy (r,s)-open set in X for each fuzzy (r,s)-open set B in Y,
(2) a *fuzzy (r,s)-semicontinuous* mapping if $f^{-1}(B)$ is a fuzzy (r,s)-semiopen set in X for each fuzzy (r,s)-open set B in Y,
(3) a *fuzzy almost (r,s)-continuous* mapping if $f^{-1}(B)$ is a fuzzy (r,s)-open set in X for each fuzzy (r,s)-regular open set B in Y,
(4) a *fuzzy weakly (r,s)-continuous* mapping if for every fuzzy (r,s)-open set B in Y, $f^{-1}(B)\subseteq \mathrm{int}(f^{-1}(\mathrm{cl}(B,r,s)),r,s)$,
(5) a *fuzzy (r,s)-irresolute* mapping if $f^{-1}(B)$ is a fuzzy (r,s)-semiopen set in X for each fuzzy (r,s)-semiopen set B in Y.

Theorem 2.10. ([9]) Let $f:(X,\mathcal{T}_1,\mathcal{T}_2)\to(Y,\mathcal{U}_1,\mathcal{U}_2)$ be a mapping from a SoIFTS X to a SoIFTS Y and $(r,s)\in I\otimes I$. Then the following statements are equivalent:

(1) f is a fuzzy almost (r,s)-continuous mapping.
(2) $f^{-1}(B)\subseteq \mathrm{int}(f^{-1}(\mathrm{int}(\mathrm{cl}(B,r,s),r,s)),r,s)$ for each fuzzy (r,s)-open set B in Y.
(3) $\mathrm{cl}(f^{-1}(\mathrm{cl}(\mathrm{int}(B,r,s),r,s)),r,s)\subseteq f^{-1}(B)$ for each fuzzy (r,s)-closed set B in Y.

3. Fuzzy weakly (r,s)-semicontinuous mappings

Now, we introduce the notion of fuzzy weakly (r,s)-semicontinuous mappings on intuitionistic fuzzy topological spaces in Šostak's sense, and then we investigate some of their properties.

THEOREM 3.1. Let $f:(X,\mathcal{T}_1,\mathcal{T}_2)\to(Y,\mathcal{U}_1,\mathcal{U}_2)$ be a mapping from a SoIFTS X to a SoIFTS Y and $(r,s)\in I\otimes I$. If f is fuzzy almost (r,s)-continuous, then f is a fuzzy weakly (r,s)-continuous mapping.

Proof. Let B be a fuzzy (r,s)-open set in Y. By Theorem 2.10, we have

$$\begin{aligned} f^{-1}(B) &\subseteq \text{int}(f^{-1}(\text{int}(\text{cl}(B,r,s),r,s)),r,s) \\ &= \text{int}(f^{-1}(\text{int}(\text{cl}(\text{cl}(B,r,s),r,s),r,s)),r,s). \end{aligned}$$

Since $\text{cl}(B,r,s)$ is a fuzzy (r,s)-semiclosed set in Y, by Theorem 2.6, we have

$$\begin{aligned} f^{-1}(B) &\subseteq \text{int}(f^{-1}(\text{int}(\text{cl}(\text{cl}(B,r,s),r,s),r,s)),r,s) \\ &\subseteq \text{int}(f^{-1}(\text{cl}(B,r,s)),r,s). \end{aligned}$$

Hence f is a fuzzy weakly (r,s)-continuous mapping. □

The converse of Theorem 3.1 need not be true(see [9]).

DEFINITION 3.2. Let $f:(X,\mathcal{T}_1,\mathcal{T}_2)\to(Y,\mathcal{U}_1,\mathcal{U}_2)$ be a mapping from a SoIFTS X to a SoIFTS Y and $(r,s)\in I\otimes I$. Then f is said to be *fuzzy weakly (r,s)-semicontinuous* if $f^{-1}(B)\subseteq \text{sint}(f^{-1}(\text{scl}(B,r,s)),r,s)$ for each fuzzy (r,s)-open set B in Y.

DEFINITION 3.3. Let $f:(X,\mathcal{T}_1,\mathcal{T}_2)\to(Y,\mathcal{U}_1,\mathcal{U}_2)$ be a mapping from a SoIFTS X to a SoIFTS Y and $(r,s)\in I\otimes I$. Then f is said to be *fuzzy weakly (r,s)-semicontinuous at an intuitionistic fuzzy point $x_{(\alpha,\beta)}$* in X if for each fuzzy (r,s)-open set B in Y such that $f(x_{(\alpha,\beta)})\in B$, there is a fuzzy (r,s)-semiopen set A in X such that $x_{(\alpha,\beta)}\in A$ and $f(A)\subseteq \text{scl}(B,r,s)$.

REMARK 3.4. It is clear that a fuzzy (r,s)-semicontinuous mapping is a fuzzy weakly (r,s)-semicontinuous mapping for each $(r,s)\in I\otimes I$. The following example shows that the converse need not be true for each $(r,s)\in I\otimes I$.

EXAMPLE 3.5. Let $X=\{x,y,z\}$ and let A_1 and A_2 be intuitionistic fuzzy sets in X defined as

$$A_1(x)=(0.3,0.6),\ A_1(y)=(0.3,0.6),\ A_1(z)=(0.3,0.3);$$

and

$$A_2(x)=(0.4,0.4),\ A_2(y)=(0.4,0.4),\ A_2(z)=(0.3,0.3).$$

Define $\mathcal{T} : I(X) \to I \otimes I$ and $\mathcal{U} : I(X) \to I \otimes I$ by

$$\mathcal{T}(A) = (\mathcal{T}_1(A), \mathcal{T}_2(A)) = \begin{cases} (1,0) & \text{if } A = \underline{0}, \underline{1}, \\ (\frac{1}{2}, \frac{1}{3}) & \text{if } A = A_2, \\ (0,1) & \text{otherwise;} \end{cases}$$

and

$$\mathcal{U}(A) = (\mathcal{U}_1(A), \mathcal{U}_2(A)) = \begin{cases} (1,0) & \text{if } A = \underline{0}, \underline{1}, \\ (\frac{1}{2}, \frac{1}{3}) & \text{if } A = A_1, A_2, \\ (0,1) & \text{otherwise.} \end{cases}$$

Then clearly $\mathcal{T}$ and $\mathcal{U}$ are SoIFTs on X. Consider a mapping $f : (X, \mathcal{T}) \to (X, \mathcal{U})$ defined by $f(x) = x$, $f(y) = y$ and $f(z) = z$. Note that

$$f^{-1}(\underline{0}) = \underline{0} \subseteq \operatorname{sint}(f^{-1}(\operatorname{scl}(\underline{0}, \frac{1}{2}, \frac{1}{3})), \frac{1}{2}, \frac{1}{3}) = \underline{0},$$

$$f^{-1}(\underline{1}) = \underline{1} \subseteq \operatorname{sint}(f^{-1}(\operatorname{scl}(\underline{1}, \frac{1}{2}, \frac{1}{3})), \frac{1}{2}, \frac{1}{3}) = \underline{1},$$

$$f^{-1}(A_1) = A_1 \subseteq \operatorname{sint}(f^{-1}(\operatorname{scl}(A_1, \frac{1}{2}, \frac{1}{3})), \frac{1}{2}, \frac{1}{3}) = A_2,$$

and

$$f^{-1}(A_2) = A_2 \subseteq \operatorname{sint}(f^{-1}(\operatorname{scl}(A_2, \frac{1}{2}, \frac{1}{3})), \frac{1}{2}, \frac{1}{3}) = A_2.$$

Hence f is fuzzy weakly $(\frac{1}{2}, \frac{1}{3})$-semicontinuous mapping. But f is not a fuzzy $(\frac{1}{2}, \frac{1}{3})$-semicontinuous mapping, because $f^{-1}(A_1) = A_1$ is not a fuzzy $(\frac{1}{2}, \frac{1}{3})$-semiopen set in $(X, \mathcal{T})$.

Theorem 3.6. Let $f : (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \mathcal{U}_1, \mathcal{U}_2)$ be a mapping from a SoIFTS X to a SoIFTS Y and $(r, s) \in I \otimes I$. If f is fuzzy almost (r, s)-continuous, then f is a fuzzy weakly (r, s)-semicontinuous mapping.

Proof. Let B be a fuzzy (r, s)-open set in Y. Then by Theorem 2.10, we have

$$\begin{aligned} f^{-1}(B) &\subseteq \operatorname{int}(f^{-1}(\operatorname{int}(\operatorname{cl}(B, r, s), r, s)), r, s) \\ &\subseteq \operatorname{sint}(f^{-1}(\operatorname{int}(\operatorname{cl}(B, r, s), r, s)), r, s) \\ &\subseteq \operatorname{sint}(f^{-1}(\operatorname{int}(\operatorname{cl}(\operatorname{scl}(B, r, s), r, s), r, s)), r, s). \end{aligned}$$

Since $\operatorname{scl}(B, r, s)$ is a fuzzy (r, s)-semiclosed set in Y, by Theorem 2.6,

$$\begin{aligned} f^{-1}(B) &\subseteq \operatorname{sint}(f^{-1}(\operatorname{int}(\operatorname{cl}(\operatorname{scl}(B, r, s), r, s), r, s)), r, s) \\ &\subseteq \operatorname{sint}(f^{-1}(\operatorname{scl}(B, r, s)), r, s). \end{aligned}$$

Thus f is a fuzzy weakly (r, s)-semicontinuous mapping. □

The following example shows that the converse of Theorem 3.6 need not be true.

EXAMPLE 3.7. Let $X = \{x, y, z\}$ and let A_1 and A_2 be intuitionistic fuzzy sets in X defined as

$$A_1(x) = (0.1, 0.3),\ A_1(y) = (0.2, 0.7),\ A_1(z) = (0.1, 0.5);$$

and

$$A_2(x) = (0.1, 0.3),\ A_2(y) = (0.3, 0.7),\ A_2(z) = (0.1, 0.5).$$

Define $\mathcal{T} : I(X) \to I \otimes I$ and $\mathcal{U} : I(X) \to I \otimes I$ by

$$\mathcal{T}(A) = (\mathcal{T}_1(A), \mathcal{T}_2(A)) = \begin{cases} (1,0) & \text{if } A = \underline{0}, \underline{1}, \\ (\frac{1}{2}, \frac{1}{3}) & \text{if } A = A_1, \\ (0,1) & \text{otherwise;} \end{cases}$$

and

$$\mathcal{U}(A) = (\mathcal{U}_1(A), \mathcal{U}_2(A)) = \begin{cases} (1,0) & \text{if } A = \underline{0}, \underline{1}, \\ (\frac{1}{2}, \frac{1}{3}) & \text{if } A = A_2, \\ (0,1) & \text{otherwise.} \end{cases}$$

Then clearly $\mathcal{T}$ and $\mathcal{U}$ are SoIFTs on X. Consider a mapping $f : (X, \mathcal{T}) \to (X, \mathcal{U})$ defined by $f(x) = x$, $f(y) = y$ and $f(z) = z$. It is easy to see that $f^{-1}(\underline{0}) = \underline{0}$, $f^{-1}(\underline{1}) = \underline{1}$, and $f^{-1}(A_2) = A_2$. Since $\underline{0}$, $\underline{1}$, and A_2 are fuzzy $(\frac{1}{2}, \frac{1}{3})$-semiopen sets in $(X, \mathcal{T})$, f is a fuzzy $(\frac{1}{2}, \frac{1}{3})$-semicontinuous mapping. Thus f is a fuzzy weakly $(\frac{1}{2}, \frac{1}{3})$-semicontinuous mapping. Since $\text{int}(\text{cl}(A_2, \frac{1}{2}, \frac{1}{3}), \frac{1}{2}, \frac{1}{3}) = A_2$, A_2 is a fuzzy $(\frac{1}{2}, \frac{1}{3})$-regular open set in $(X, \mathcal{U})$. But f is not a fuzzy almost $(\frac{1}{2}, \frac{1}{3})$-continuous mapping, because $f^{-1}(A_2) = A_2$ is not a fuzzy $(\frac{1}{2}, \frac{1}{3})$-open set in $(X, \mathcal{T})$.

The following examples show that a fuzzy weakly (r, s)-semicontinuous mapping need not be fuzzy weakly (r, s)-continuous, and vice versa for each $(r, s) \in I \otimes I$.

EXAMPLE 3.8. Let $X = \{x, y, z\}$ and let A_1, A_2 and A_3 be intuitionistic fuzzy sets in X defined as

$$A_1(x) = (0.1, 0.7),\ A_1(y) = (0.1, 0.8),\ A_1(z) = (0, 1);$$

$$A_2(x) = (0.2, 0.5),\ A_2(y) = (0.3, 0.6),\ A_2(z) = (0.3, 0.6);$$

and

$$A_3(x) = (0.4, 0.4),\ A_3(y) = (0.4, 0.4),\ A_3(z) = (0.3, 0.3).$$

Define $\mathcal{T} : I(X) \to I \otimes I$ and $\mathcal{U} : I(X) \to I \otimes I$ by

$$\mathcal{T}(A) = (\mathcal{T}_1(A), \mathcal{T}_2(A)) = \begin{cases} (1,0) & \text{if } A = \underline{0}, \underline{1}, \\ (\frac{1}{2}, \frac{1}{3}) & \text{if } A = A_1, \\ (0,1) & \text{otherwise;} \end{cases}$$

and

$$\mathcal{U}(A) = (\mathcal{U}_1(A), \mathcal{U}_2(A)) = \begin{cases} (1,0) & \text{if } A = \underline{0}, \underline{1}, \\ (\frac{1}{2}, \frac{1}{3}) & \text{if } A = A_2, A_3, \\ (0,1) & \text{otherwise.} \end{cases}$$

Then clearly $\mathcal{T}$ and $\mathcal{U}$ are SoIFTs on X. Consider a mapping $f : (X, \mathcal{T}) \to (X, \mathcal{U})$ defined by $f(x) = x$, $f(y) = y$ and $f(z) = z$. Note that

$$f^{-1}(\underline{0}) = \underline{0} \subseteq \text{sint}(f^{-1}(\text{scl}(\underline{0}, \frac{1}{2}, \frac{1}{3})), \frac{1}{2}, \frac{1}{3}) = \underline{0},$$

$$f^{-1}(\underline{1}) = \underline{1} \subseteq \text{sint}(f^{-1}(\text{scl}(\underline{1}, \frac{1}{2}, \frac{1}{3})), \frac{1}{2}, \frac{1}{3}) = \underline{1},$$

$$f^{-1}(A_2) = A_2 \subseteq \text{sint}(f^{-1}(\text{scl}(A_2, \frac{1}{2}, \frac{1}{3})), \frac{1}{2}, \frac{1}{3}) = A_3,$$

and

$$f^{-1}(A_3) = A_3 \subseteq \text{sint}(f^{-1}(\text{scl}(A_3, \frac{1}{2}, \frac{1}{3})), \frac{1}{2}, \frac{1}{3}) = A_3.$$

Thus f is a fuzzy weakly $(\frac{1}{2}, \frac{1}{3})$-semicontinuous mapping. But f is not a fuzzy weakly $(\frac{1}{2}, \frac{1}{3})$-continuous mapping, because

$$f^{-1}(A_2) = A_2 \nsubseteq \text{int}(f^{-1}(\text{cl}(A_2, \frac{1}{2}, \frac{1}{3})), \frac{1}{2}, \frac{1}{3}) = A_1.$$

Example 3.9. Let $X = \{x, y, z\}$ and let A_1 and A_2 be intuitionistic fuzzy sets in X defined as

$$A_1(x) = (0.6, 0.3),\ A_1(y) = (0.2, 0.5),\ A_1(z) = (0.4, 0.4);$$

and

$$A_2(x) = (0.2, 0.7),\ A_2(y) = (0.2, 0.7),\ A_2(z) = (0.3, 0.6).$$

Define $\mathcal{T} : I(X) \to I \otimes I$ and $\mathcal{U} : I(X) \to I \otimes I$ by

$$\mathcal{T}(A) = (\mathcal{T}_1(A), \mathcal{T}_2(A)) = \begin{cases} (1,0) & \text{if } A = \underline{0}, \underline{1}, \\ (\frac{1}{2}, \frac{1}{3}) & \text{if } A = A_1, \\ (0,1) & \text{otherwise;} \end{cases}$$

and

$$\mathcal{U}(A) = (\mathcal{U}_1(A), \mathcal{U}_2(A)) = \begin{cases} (1,0) & \text{if } A = \underline{0}, \underline{1}, \\ (\frac{1}{2}, \frac{1}{3}) & \text{if } A = A_2, \\ (0,1) & \text{otherwise.} \end{cases}$$

Then clearly $\mathcal{T}$ and $\mathcal{U}$ are SoIFTs on X. Consider a mapping $f : (X, \mathcal{T}) \to (X, \mathcal{U})$ defined by $f(x) = x$, $f(y) = y$ and $f(z) = z$. Note that

$$f^{-1}(\underline{0}) = \underline{0} \subseteq \text{int}(f^{-1}(\text{cl}(\underline{0}, \frac{1}{2}, \frac{1}{3})), \frac{1}{2}, \frac{1}{3}) = \underline{0},$$

$$f^{-1}(\underline{1}) = \underline{1} \subseteq \operatorname{int}(f^{-1}(\operatorname{cl}(\underline{1}, \frac{1}{2}, \frac{1}{3})), \frac{1}{2}, \frac{1}{3}) = \underline{1},$$

and

$$f^{-1}(A_2) = A_2 \subseteq \operatorname{int}(f^{-1}(\operatorname{cl}(A_2, \frac{1}{2}, \frac{1}{3})), \frac{1}{2}, \frac{1}{3}) = A_1,$$

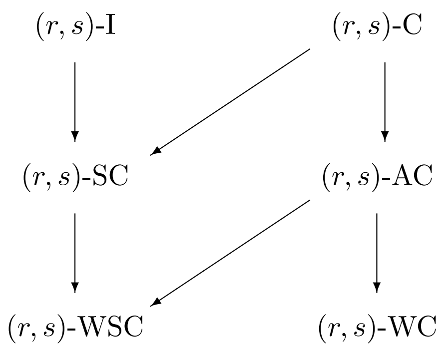
Thus f is a fuzzy weakly $(\frac{1}{2}, \frac{1}{3})$-continuous mapping. But f is not a fuzzy weakly $(\frac{1}{2}, \frac{1}{3})$-semicontinuous mapping, because

$$f^{-1}(A_2) = A_2 \nsubseteq \operatorname{sint}(f^{-1}(\operatorname{scl}(A_2, \frac{1}{2}, \frac{1}{3})), \frac{1}{2}, \frac{1}{3}) = \underline{0}.$$

From the above two examples, we have the following result.

THEOREM 3.10. Fuzzy weakly (r, s)-semicontinuous mapping and fuzzy weakly (r, s)-continuous mapping are independent notions.

REMARK 3.11. Using above definitions, theorems, and results of [9], we give the following implication diagram to indicate the relations among the different notions of fuzzy (r, s)-continuous((r, s)-C), fuzzy (r, s)-irresolute((r, s)-I), fuzzy (r, s)-semicontinuous((r, s)-SC), fuzzy almost (r, s)-continuous((r, s)-AC), fuzzy weakly (r, s)-continuous((r, s)-WC), and fuzzy weakly (r, s)-semicontinuous((r, s)-WSC) mapping. None of the undrawn implicaions holds.



THEOREM 3.12. Let $f : (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \mathcal{U}_1, \mathcal{U}_2)$ be a mapping from a SoIFTS X to a SoIFTS Y and $(r, s) \in I \otimes I$. Then the following statements are equivalent:

(1) f is a fuzzy weakly (r, s)-semicontinuous mapping.
(2) $\operatorname{scl}(f^{-1}(\operatorname{sint}(B, r, s)), r, s) \subseteq f^{-1}(B)$ for each fuzzy (r, s)-closed set B in Y.
(3) $f^{-1}(\operatorname{int}(B, r, s)) \subseteq \operatorname{sint}(f^{-1}(\operatorname{scl}(B, r, s)), r, s)$ for each intuitionistic fuzzy set B in Y.
(4) $\operatorname{scl}(f^{-1}(\operatorname{sint}(B, r, s)), r, s) \subseteq f^{-1}(\operatorname{cl}(B, r, s))$ for each intuitionistic fuzzy set B in Y.

Proof. (1) ⇒ (2) Let f be a fuzzy weakly (r, s)-semicontinuous mapping and B a fuzzy (r, s)-closed set in Y. Since B^c is a fuzzy (r, s)-open set in Y,

$$\begin{aligned}(f^{-1}(B))^c = f^{-1}(B^c) &\subseteq \operatorname{sint}(f^{-1}(\operatorname{scl}(B^c, r, s)), r, s)\\ &= \operatorname{scl}(f^{-1}(\operatorname{sint}(B, r, s)), r, s)^c.\end{aligned}$$

Thus we have $\operatorname{scl}(f^{-1}(\operatorname{sint}(B, r, s)), r, s) \subseteq f^{-1}(B)$.

(2) ⇒ (1) Let B be a fuzzy (r, s)-open set in Y. Then B^c is a fuzzy (r, s)-closed set in Y. By (2),

$$\begin{aligned}(f^{-1}(B))^c = f^{-1}(B^c) &\supseteq \operatorname{scl}(f^{-1}(\operatorname{sint}(B^c, r, s)), r, s)\\ &= \operatorname{sint}(f^{-1}(\operatorname{scl}(B, r, s)), r, s)^c.\end{aligned}$$

Thus we have $f^{-1}(B) \subseteq \operatorname{sint}(f^{-1}(\operatorname{scl}(B, r, s)), r, s)$. Hence f is a fuzzy weakly (r, s)-semicontinuous mapping.

(1) ⇒ (3) Let B be an intuitionistic fuzzy set in Y. Then $\operatorname{int}(B, r, s)$ is a fuzzy (r, s)-open set in Y. Since f is a fuzzy weakly (r, s)-continuous mapping,

$$\begin{aligned}f^{-1}(\operatorname{int}(B, r, s)) &\subseteq \operatorname{sint}(f^{-1}(\operatorname{scl}(\operatorname{int}(B, r, s), r, s)), r, s)\\ &\subseteq \operatorname{sint}(f^{-1}(\operatorname{scl}(B, r, s)), r, s).\end{aligned}$$

(3) ⇒ (1) It is obvious.

(2) ⇒ (4) Let B be an intuitionistic fuzzy set in Y. Then $\operatorname{cl}(B, r, s)$ is a fuzzy (r, s)-closed set in Y. By (2),

$$\begin{aligned}\operatorname{scl}(f^{-1}(\operatorname{sint}(B, r, s)), r, s) &\subseteq \operatorname{scl}(f^{-1}(\operatorname{sint}(\operatorname{cl}(B, r, s), r, s)), r, s)\\ &\subseteq f^{-1}(\operatorname{cl}(B, r, s)).\end{aligned}$$

(4) ⇒ (2) It is obvious. □

Theorem 3.13. Let $f : (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \mathcal{U}_1, \mathcal{U}_2)$ be a mapping from a SoIFTS X to a SoIFTS Y and $(r, s) \in I \otimes I$. Then f is a fuzzy weakly (r, s)-semicontinuous mapping if and only if f is fuzzy weakly (r, s)-semicontinuous for each intuitionistic fuzzy point $x_{(\alpha,\beta)}$ in X.

Proof. Let f be a fuzzy weakly (r, s)-semicontinuous mapping, $x_{(\alpha,\beta)}$ an intuitionistic fuzzy point in X, and B a fuzzy (r, s)-open set in Y such that $f(x_{(\alpha,\beta)}) \in B$. Then $x_{(\alpha,\beta)} \in f^{-1}(B) \subseteq \operatorname{sint}(f^{-1}(\operatorname{scl}(B, r, s)), r, s)$. Let $A = \operatorname{sint}(f^{-1}(\operatorname{scl}(B, r, s)), r, s)$. Then A is a fuzzy (r, s)-semiopen set in X and

$$\begin{aligned}f(A) &= f(\operatorname{sint}(f^{-1}(\operatorname{scl}(B, r, s)), r, s))\\ &\subseteq f(f^{-1}(\operatorname{scl}(B, r, s))) \subseteq \operatorname{scl}(B, r, s).\end{aligned}$$

Hence f is fuzzy weakly (r, s)-semicontinuous at $x_{(\alpha,\beta)}$. Therefore f is fuzzy weakly (r, s)-semicontinuous for each intuitionistic fuzzy point in X, because $x_{(\alpha,\beta)}$ is an arbitrary intuitionistic fuzzy point in X.
Conversely, let B be a fuzzy (r, s)-open set in Y and $x_{(\alpha,\beta)}$ an intuitionistic fuzzy point such that $x_{(\alpha,\beta)} \in f^{-1}(B)$. Then $f(x_{(\alpha,\beta)}) \in B$. By hypothesis, there is a fuzzy (r, s)-semiopen set $A_{x_{(\alpha,\beta)}}$ in X such that $x_{(\alpha,\beta)} \in A_{x_{(\alpha,\beta)}}$ and $f(A_{x_{(\alpha,\beta)}}) \subseteq \mathrm{scl}(B, r, s)$. Thus

$$x_{(\alpha,\beta)} \in A_{x_{(\alpha,\beta)}} \subseteq f^{-1}(f(A_{x_{(\alpha,\beta)}})) \subseteq f^{-1}(\mathrm{scl}(B, r, s)).$$

So,

$$\begin{aligned} x_{(\alpha,\beta)} \in A_{x_{(\alpha,\beta)}} &= \mathrm{sint}(A_{x_{(\alpha,\beta)}}, r, s) \\ &\subseteq \mathrm{sint}(f^{-1}(\mathrm{scl}(B, r, s)), r, s). \end{aligned}$$

Hence

$$\begin{aligned} f^{-1}(B) &= \bigcup\{x_{(\alpha,\beta)} \mid x_{(\alpha,\beta)} \in f^{-1}(B)\} \\ &\subseteq \bigcup\{A_{x_{(\alpha,\beta)}} \mid x_{(\alpha,\beta)} \in f^{-1}(B)\} \\ &\subseteq \mathrm{sint}(f^{-1}(\mathrm{scl}(B, r, s)), r, s). \end{aligned}$$

Therefore f is a fuzzy weakly (r, s)-semicontinuous mapping. □

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