

LINEAR θ -DERIVATIONS ON JB^* -TRIPLES

CHUNKIL BAK*

ABSTRACT. In [1], the concept of generalized (θ, ϕ) -derivations on rings was introduced. We introduce the concept of linear θ -derivations on JB^* -triples, and prove the Cauchy–Rassias stability of linear θ -derivations on JB^* -triples.

1. Introduction

The original motivation to introduce the class of nonassociative algebras known as Jordan algebras came from quantum mechanics (see [24]). Let $\mathcal{H}$ be a complex Hilbert space, regarded as the “state space” of a quantum mechanical system. Let $\mathcal{L}(\mathcal{H})$ be the real vector space of all bounded self-adjoint linear operators on $\mathcal{H}$, interpreted as the (bounded) *observables* of the system. In 1932, Jordan observed that $\mathcal{L}(\mathcal{H})$ is a (nonassociative) algebra via the *anticommutator product* $x \circ y := \frac{xy+yx}{2}$. A commutative algebra X with product $x \circ y$ (not necessarily given by an anticommutator) is called a *Jordan algebra* if $x^2 \circ (x \circ y) = x \circ (x^2 \circ y)$ holds.

A complex Jordan algebra $\mathcal{B}$ with product $x \circ y$, unit element e and involution $x \mapsto x^*$ is called a JB^* -algebra if $\mathcal{B}$ carries a Banach space norm $\|\cdot\|$ satisfying $\|x \circ y\| \leq \|x\| \cdot \|y\|$ and $\|\{xx^*x\}\| = \|x\|^3$. Here $\{xy^*z\} := x \circ (y^* \circ z) - y^* \circ (z \circ x) + z \circ (x \circ y^*)$ denotes the *Jordan triple product* of $x, y, z \in \mathcal{B}$.

The Jordan triple product of a JB^* -algebra leads to a more general algebraic structure, which turns out to be appropriate for most applications to analysis: the so-called Jordan triple systems. Suppose $\mathcal{J}$ is a complex vector space endowed with a real trilinear composition

$$\mathcal{J} \times \mathcal{J} \times \mathcal{J} \ni (x, y, z) \mapsto \{xy^*z\} \in \mathcal{J}$$

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which is complex bilinear in (x, z) and conjugate linear in y . Then $\mathcal{J}$ is called a *Jordan triple system* if $\{xy^*z\} = \{zy^*x\}$ and

$$\{\{xy^*z\}u^*v\} + \{\{xy^*v\}u^*z\} - \{xy^*\{zu^*v\}\} = \{z\{yx^*u\}^*v\}$$

hold.

We are interested in Jordan triple systems having a Banach space structure. A complex Jordan triple system $\mathcal{J}$ with a Banach space norm $\|\cdot\|$ is called a *J^* -triple* if, for every $x \in \mathcal{J}$, the operator $x \square x^*$ is hermitian in the sense of Banach algebra theory. Here the operator $x \square x^*$ on $\mathcal{J}$ is defined by $(x \square x^*)y := \{xx^*y\}$. This implies that $x \square x^*$ has real spectrum $\sigma(x \square x^*) \subset \mathbb{R}$. A J^* -triple $\mathcal{J}$ is called a *JB^* -triple* if every $x \in \mathcal{J}$ satisfies $\sigma(x \square x^*) \geq 0$ and $\|x \square x^*\| = \|x\|^2$.

Our knowledge concerning the continuity properties of epimorphisms on Banach algebras, Jordan-Banach algebras, and, more generally, nonassociative complete normed algebras, is now fairly complete and satisfactory (see [6], [25]). A basic continuity problem consists in determining algebraic conditions on a Banach algebra A which ensure that derivations on A are continuous. In 1996, Villena [25] proved that derivations on semisimple Jordan-Banach algebras are continuous.

Let X and Y be Banach spaces with norms $\|\cdot\|$ and $\|\cdot\|$, respectively. Consider $f : X \rightarrow Y$ to be a mapping such that $f(tx)$ is continuous in $t \in \mathbb{R}$ for each fixed $x \in X$. Th.M. Rassias [19] introduced the following inequality, that we call *Cauchy–Rassias inequality*: Assume that there exist constants $\epsilon \geq 0$ and $p \in [0, 1)$ such that

$$\|f(x+y) - f(x) - f(y)\| \leq \epsilon(\|x\|^p + \|y\|^p)$$

for all $x, y \in X$. Th.M. Rassias [19] showed that there exists a unique $\mathbb{R}$ -linear mapping $T : X \rightarrow Y$ such that

$$\|f(x) - T(x)\| \leq \frac{2\epsilon}{2-2^p} \|x\|^p$$

for all $x \in X$. Beginning around the year 1980 the topic of approximate homomorphisms, or the stability of the equation of homomorphism, was

studied by a number of mathematicians. Găvruta [3] generalized the Rassias' result in the following form: Let G be an abelian group and X a Banach space. Denote by $\varphi : G \times G \rightarrow [0, \infty)$ a function such that

$$\tilde{\varphi}(x, y) = \sum_{k=0}^{\infty} \frac{1}{2^k} \varphi(2^k x, 2^k y) < \infty$$

for all $x, y \in G$. Suppose that $f : G \rightarrow X$ is a mapping satisfying

$$\|f(x+y) - f(x) - f(y)\| \leq \varphi(x, y)$$

for all $x, y \in G$. Then there exists a unique additive mapping $T : G \rightarrow X$ such that

$$\|f(x) - T(x)\| \leq \frac{1}{2} \tilde{\varphi}(x, x)$$

for all $x \in G$.

Jun and Lee [5] proved the following: Denote by $\varphi : X \setminus \{0\} \times X \setminus \{0\} \rightarrow [0, \infty)$ a function such that

$$\tilde{\varphi}(x, y) = \sum_{j=0}^{\infty} \frac{1}{3^j} \varphi(3^j x, 3^j y) < \infty$$

for all $x, y \in X \setminus \{0\}$. Suppose that $f : X \rightarrow Y$ is a mapping satisfying

$$\|2f\left(\frac{x+y}{2}\right) - f(x) - f(y)\| \leq \varphi(x, y)$$

for all $x, y \in X \setminus \{0\}$. Then there exists a unique additive mapping $T : X \rightarrow Y$ such that

$$\|f(x) - f(0) - T(x)\| \leq \frac{1}{3} (\tilde{\varphi}(x, -x) + \tilde{\varphi}(-x, 3x))$$

for all $x \in X \setminus \{0\}$. The stability problem of functional equations has been investigated in several papers (see [7]–[23]).

Let θ, ϕ be endomorphisms of a ring R . An additive mapping $D : R \rightarrow R$ is called a (θ, ϕ) -derivation on R if $D(xy) = D(x)\theta(y) + \phi(x)D(y)$ holds for all $x, y \in R$. An additive mapping $U : R \rightarrow R$ is called a *generalized* (θ, ϕ) -derivation on R if there exists a (θ, ϕ) -derivation $D : R \rightarrow R$ such that $U(xy) = U(x)\theta(y) + \phi(x)D(y)$ holds for all $x, y \in R$ (see [1], [2], [4]).

In this paper, we introduce the concept of linear θ -derivations on JB^* -triples, and prove the Cauchy–Rassias stability of linear θ -derivations on JB^* -triples.

2. Linear θ -derivations on JB^* -triples

Throughout this section, let B be a complex JB^* -triple with norm $\|\cdot\|$.

DEFINITION 2.1. Let $\theta : B \rightarrow B$ be an involutive $\mathbb{C}$ -linear mapping. An involutive $\mathbb{C}$ -linear mapping $D : B \rightarrow B$ is called a *linear θ -derivation* on B if

$$D(\{xy^*z\}) = \{D(x)\theta(y)^*\theta(z)\} + \{\theta(x)D(y)^*\theta(z)\} + \{\theta(x)\theta(y)^*D(z)\}$$

holds for all $x, y, z \in B$.

THEOREM 2.1. Let $f, h : B \rightarrow B$ be mappings with $f(0) = h(0) = 0$ for which there exists a function $\varphi : B^3 \rightarrow [0, \infty)$ such that

$$(2.1) \quad \tilde{\varphi}(x, y, z) := \sum_{j=0}^{\infty} \frac{1}{2^j} \varphi(2^j x, 2^j y, 2^j z) < \infty,$$

$$(2.2) \quad \|f(\mu x + \mu y + z^*) - \mu f(x) - \mu f(y) - f(z)^*\| \leq \varphi(x, y, z),$$

$$(2.3) \quad \|h(\mu x + \mu y + z^*) - \mu h(x) - \mu h(y) - h(z)^*\| \leq \varphi(x, y, z),$$

$$(2.4) \quad \begin{aligned} & \|f(\{xy^*z\}) - \{f(x)h(y)^*h(z)\} - \{h(x)f(y)^*h(z)\} \\ & \quad - \{h(x)h(y)^*f(z)\}\| \leq \varphi(x, y, z), \end{aligned}$$

for all $x, y, z \in B$ and all $\mu \in S^1 := \{\lambda \in \mathbb{C} \mid |\lambda| = 1\}$. Then there exist unique involutive $\mathbb{C}$ -linear mappings $D, \theta : B \rightarrow B$ such that

$$(2.5) \quad \|f(x) - D(x)\| \leq \frac{1}{2} \tilde{\varphi}(x, x, 0),$$

$$(2.6) \quad \|h(x) - \theta(x)\| \leq \frac{1}{2} \tilde{\varphi}(x, x, 0)$$

for all $x \in B$. Moreover, $D : B \rightarrow B$ is a linear θ -derivation on B .

Proof. Let $\mu = 1 \in S^1$ and $z = 0$ in (2.2) and (2.3). It follows from the Găvruta's theorem [3] that there exist unique additive mappings $D, \theta : B \rightarrow B$ satisfying (2.5) and (2.6). The additive mappings $D, \theta : B \rightarrow B$ are given by

$$(2.7) \quad D(x) = \lim_{l \rightarrow \infty} \frac{1}{2^l} f(2^l x),$$

$$(2.8) \quad \theta(x) = \lim_{l \rightarrow \infty} \frac{1}{2^l} h(2^l x)$$

for all $x \in B$.

By the same reasoning as in the proof of Theorem 2.1 in [12], the additive mappings $D, \theta : B \rightarrow B$ are $\mathbb{C}$ -linear.

Letting $x = y = 0$ in (2.2), we get

$$D(x^*) = \lim_{l \rightarrow \infty} \frac{f(2^l x^*)}{2^l} = \lim_{l \rightarrow \infty} \frac{(f(2^l x))^*}{2^l} = \left(\lim_{l \rightarrow \infty} \frac{f(2^l x)}{2^l} \right)^* = D(x)^*$$

for all $x \in B$.

Similarly, one can show that $\theta : B \rightarrow B$ is an involutive mapping.

It follows from (2.4) that

$$\begin{aligned} \frac{1}{2^{3l}} \|f(2^{3l}\{xy^*z\}) - \{f(2^l x)h(2^l y)^*h(2^l z)\} - \{h(2^l x)f(2^l y)^*h(2^l z)\} \\ - \{h(2^l x)h(2^l y)^*f(2^l z)\}\| \leq \frac{1}{2^{3l}} \varphi(2^l x, 2^l y, 2^l z) \leq \frac{1}{2^l} \varphi(2^l x, 2^l y, 2^l z), \end{aligned}$$

which tends to zero as $l \rightarrow \infty$ for all $x, y, z \in B$ by (2.1). By (2.7) and (2.8),

$$D(\{xy^*z\}) = \{D(x)\theta(y)^*\theta(z)\} + \{\theta(x)D(y)^*\theta(z)\} + \{\theta(x)\theta(y)^*D(z)\}$$

for all $x, y, z \in B$. So the additive mapping $D : B \rightarrow B$ is a linear θ -derivation on B , as desired. $\square$

COROLLARY 2.2. *Let $f, h : B \rightarrow B$ be mappings with $f(0) = h(0) = 0$ for which there exist constants $\epsilon \geq 0$ and $p \in [0, 1)$ such that*

$$\begin{aligned} \|f(\mu x + \mu y + z^*) - \mu f(x) - \mu f(y) - f(z)^*\| &\leq \epsilon(\|x\|^p + \|y\|^p + \|z\|^p), \\ \|h(\mu x + \mu y + z^*) - \mu h(x) - \mu h(y) - h(z)^*\| &\leq \epsilon(\|x\|^p + \|y\|^p + \|z\|^p), \\ \|f(\{xy^*z\}) - \{f(x)h(y)^*h(z)\} - \{h(x)f(y)^*h(z)\} \\ &\quad - \{h(x)h(y)^*f(z)\}\| \leq \epsilon(\|x\|^p + \|y\|^p + \|z\|^p) \end{aligned}$$

for all $x, y, z \in B$ and all $\mu \in S^1$. Then there exist unique involutive $\mathbb{C}$ -linear mappings $D, \theta : B \rightarrow B$ such that

$$\begin{aligned} \|f(x) - D(x)\| &\leq \frac{2\epsilon}{2 - 2^p} \|x\|^p, \\ \|h(x) - \theta(x)\| &\leq \frac{2\epsilon}{2 - 2^p} \|x\|^p \end{aligned}$$

for all $x \in B$. Moreover, $D : B \rightarrow B$ is a linear θ -derivation on B .

Proof. Define $\varphi(x, y, z) = \epsilon(\|x\|^p + \|y\|^p + \|z\|^p)$ to be Th.M. Rassias upper bound in the Cauchy–Rassias inequality, and apply Theorem 2.1. $\square$

THEOREM 2.3. Let $f, h : B \rightarrow B$ be mappings with $f(0) = h(0) = 0$ for which there exists a function $\varphi : B^3 \rightarrow [0, \infty)$ satisfying (2.4) such that

$$(2.9) \quad \tilde{\varphi}(x, y, z) := \sum_{j=0}^{\infty} \frac{1}{3^j} \varphi(3^j x, 3^j y, 3^j z) < \infty,$$

$$(2.10) \quad \|2f(\frac{\mu x + \mu y + z^*}{2}) - \mu f(x) - \mu f(y) - f(z)^*\| \leq \varphi(x, y, z),$$

$$(2.11) \quad \|2h(\frac{\mu x + \mu y + z^*}{2}) - \mu h(x) - \mu h(y) - h(z)^*\| \leq \varphi(x, y, z)$$

for all $x, y, z \in B$ and all $\mu \in S^1$. Then there exist unique involutive $\mathbb{C}$ -linear mappings $D, \theta : B \rightarrow B$ such that

$$(2.12) \quad \|f(x) - D(x)\| \leq \frac{1}{3} (\tilde{\varphi}(x, -x, 0) + \tilde{\varphi}(-x, 3x, 0)),$$

$$(2.13) \quad \|h(x) - \theta(x)\| \leq \frac{1}{3} (\tilde{\varphi}(x, -x, 0) + \tilde{\varphi}(-x, 3x, 0))$$

for all $x \in B$. Moreover, $D : B \rightarrow B$ is a linear θ -derivation on B .

Proof. Let $z = 0$ in (2.10) and (2.11). It follows from the Jun and Lee's theorem [5, Theorem 1] that there exist unique additive mappings $D, \theta : B \rightarrow B$ satisfying (2.12) and (2.13). The additive mappings $D, \theta : B \rightarrow B$ are given by

$$(2.14) \quad D(x) = \lim_{l \rightarrow \infty} \frac{1}{3^l} f(3^l x),$$

$$(2.15) \quad \theta(x) = \lim_{l \rightarrow \infty} \frac{1}{3^l} h(3^l x)$$

for all $x \in B$.

The rest of the proof is similar to the proof of Theorem 2.1. $\square$

COROLLARY 2.4. Let $f, h : B \rightarrow B$ be mappings with $f(0) = h(0) = 0$ for which there exist constants $\epsilon \geq 0$ and $p \in [0, 1)$ such that

$$\begin{aligned} \|2f(\frac{\mu x + \mu y + z^*}{2}) - \mu f(x) - \mu f(y) - f(z)^*\| &\leq \epsilon(\|x\|^p + \|y\|^p + \|z\|^p), \\ \|2h(\frac{\mu x + \mu y + z^*}{2}) - \mu h(x) - \mu h(y) - h(z)^*\| &\leq \epsilon(\|x\|^p + \|y\|^p + \|z\|^p), \\ \|f(\{xy^*z\}) - \{f(x)h(y)^*h(z)\} - \{h(x)f(y)^*h(z)\} \\ &\quad - \{h(x)h(y)^*f(z)\}\| \leq \epsilon(\|x\|^p + \|y\|^p + \|z\|^p) \end{aligned}$$

for all $x, y, z \in B$ and all $\mu \in S^1$. Then there exist involutive $\mathbb{C}$ -linear mappings $D, \theta : B \rightarrow B$ such that

$$\begin{aligned} \|f(x) - D(x)\| &\leq \frac{3 + 3^p}{3 - 3^p} \epsilon \|x\|^p, \\ \|h(x) - \theta(x)\| &\leq \frac{3 + 3^p}{3 - 3^p} \epsilon \|x\|^p \end{aligned}$$

for all $x \in B$. Moreover, $D : B \rightarrow B$ is a linear θ -derivation on B .

Proof. Define $\varphi(x, y, z) = \epsilon(\|x\|^p + \|y\|^p + \|z\|^p)$, and apply Theorem 2.3, as desired. $\square$

THEOREM 2.5. Let $f, h : B \rightarrow B$ be mappings with $f(0) = h(0) = 0$ for which there exists a function $\varphi : B^3 \rightarrow [0, \infty)$ satisfying (2.10), (2.11) and (2.4) such that

$$(2.16) \quad \sum_{j=0}^{\infty} 3^{3j} \varphi\left(\frac{x}{3^j}, \frac{y}{3^j}, \frac{z}{3^j}\right) < \infty$$

for all $x, y, z \in B$. Then there exist unique involutive $\mathbb{C}$ -linear mappings $D, \theta : B \rightarrow B$ such that

$$(2.17) \quad \|f(x) - D(x)\| \leq \tilde{\varphi}\left(\frac{x}{3}, -\frac{x}{3}, 0\right) + \tilde{\varphi}\left(-\frac{x}{3}, x, 0\right),$$

$$(2.18) \quad \|h(x) - \theta(x)\| \leq \tilde{\varphi}\left(\frac{x}{3}, -\frac{x}{3}, 0\right) + \tilde{\varphi}\left(-\frac{x}{3}, x, 0\right)$$

for all $x \in B$, where

$$\tilde{\varphi}(x, y, z) := \sum_{j=0}^{\infty} 3^j \varphi\left(\frac{x}{3^j}, \frac{y}{3^j}, \frac{z}{3^j}\right)$$

for all $x, y, z \in B$. Moreover, $D : B \rightarrow B$ is a linear θ -derivation on B .

Proof. By the Jun and Lee's theorem [5, Theorem 6], it follows from (2.16), (2.10) and (2.11) that there exist unique additive mappings $D, \theta : B \rightarrow B$ satisfying (2.17) and (2.18). The additive mappings $D, \theta : B \rightarrow B$ are given by

$$(2.19) \quad D(x) = \lim_{l \rightarrow \infty} 3^l f\left(\frac{x}{3^l}\right),$$

$$(2.20) \quad \theta(x) = \lim_{l \rightarrow \infty} 3^l h\left(\frac{x}{3^l}\right)$$

for all $x \in B$.

By a similar method to the proof of Theorem 2.1, one can show that $D, \theta : B \rightarrow B$ are involutive $\mathbb{C}$ -linear mappings.

It follows from (2.4) that

$$3^{3l} \left\| f\left(\frac{\{xy^*z\}}{3^{3l}}\right) - \left\{f\left(\frac{x}{3^l}\right)h\left(\frac{y}{3^l}\right)^*h\left(\frac{z}{3^l}\right)\right\} - \left\{h\left(\frac{x}{3^l}\right)f\left(\frac{y}{3^l}\right)^*h\left(\frac{z}{3^l}\right)\right\} - \left\{h\left(\frac{x}{3^l}\right)h\left(\frac{y}{3^l}\right)^*f\left(\frac{z}{3^l}\right)\right\} \right\| \leq 3^{3l} \varphi\left(\frac{x}{3^l}, \frac{y}{3^l}, \frac{z}{3^l}\right),$$

which tends to zero as $l \rightarrow \infty$ for all $x, y, z \in B$ by (2.16). By (2.19) and (2.20),

$$D(\{xy^*z\}) = \{D(x)\theta(y)^*\theta(z)\} + \{\theta(x)D(y)^*\theta(z)\} + \{\theta(x)\theta(y)^*D(z)\}$$

for all $x, y, z \in B$. So the additive mapping $D : B \rightarrow B$ is a linear θ -derivation on B , as desired. $\square$

COROLLARY 2.6. *Let $f, h : B \rightarrow B$ be mappings with $f(0) = h(0) = 0$ for which there exist constants $\epsilon \geq 0$ and $p \in (3, \infty)$ such that*

$$\begin{aligned} \|2f\left(\frac{\mu x + \mu y + z^*}{2}\right) - \mu f(x) - \mu f(y) - f(z)^*\| &\leq \epsilon(\|x\|^p + \|y\|^p + \|z\|^p), \\ \|2h\left(\frac{\mu x + \mu y + z^*}{2}\right) - \mu h(x) - \mu h(y) - h(z)^*\| &\leq \epsilon(\|x\|^p + \|y\|^p + \|z\|^p), \\ \|f(\{xy^*z\}) - \{f(x)h(y)^*h(z)\} - \{h(x)f(y)^*h(z)\} \\ &\quad - \{h(x)h(y)^*f(z)\}\| \leq \epsilon(\|x\|^p + \|y\|^p + \|z\|^p) \end{aligned}$$

for all $x, y, z \in B$ and all $\mu \in S^1$. Then there exist unique involutive $\mathbb{C}$ -linear mappings $D, \theta : B \rightarrow B$ such that

$$\begin{aligned} \|f(x) - D(x)\| &\leq \frac{3^p + 3}{3^p - 3} \epsilon \|x\|^p, \\ \|h(x) - \theta(x)\| &\leq \frac{3^p + 3}{3^p - 3} \epsilon \|x\|^p \end{aligned}$$

for all $x \in B$. Moreover, $D : B \rightarrow B$ is a linear θ -derivation on B .

Proof. Define $\varphi(x, y, z) = \epsilon(\|x\|^p + \|y\|^p + \|z\|^p)$, and apply Theorem 2.5, as desired. $\square$

DEFINITION 2.2. Let $\theta : B \rightarrow B$ be an involutive $\mathbb{C}$ -linear mapping. An involutive $\mathbb{C}$ -linear mapping $D : B \rightarrow B$ is called a *Jordan linear θ -derivation* on B if

$$D(\{xx^*x\}) = \{D(x)\theta(x)^*\theta(x)\} + \{\theta(x)D(x)^*\theta(x)\} + \{\theta(x)\theta(x)^*D(x)\}$$

holds for all $x \in B$.

Problem 2.1. Is every Jordan linear θ -derivation a linear θ -derivation?

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DEPARTMENT OF MATHEMATICS
CHUNGNAM NATIONAL UNIVERSITY
DAEJEON 305–764, KOREA
E-mail: cbak@math.cnu.ac.kr