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FIXED POINT THEOREMS FOR MÖNCH TYPE MAPS IN ABSTRACT CONVEX UNIFORM SPACES

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Abstract. In this paper, first, we present new fixed point theorems for Mönch type multimaps on abstract convex uniform spaces and, also, a fixed point theorem for Mönch type multimaps in Hausdorff KKM $L\Gamma$-spaces. Second, we show that Mönch type multimaps in the better admissible class defined on an $L\Gamma$-space have fixed point properties whenever their ranges are Klee approximable. Finally, we obtain fixed point theorems on $\mathfrak{KC}$-maps whose ranges are Φ-sets.

1. Introduction and preliminaries

Relaxing compactness of multimaps in fixed point theory is an important task. O'Regan and Precup [6] have mitigated the compactness such as a fixed point theorem for Mönch type mutimaps in Banach space. Huang et al. [2] presents some fixed point results for Mönch type self-multimaps with s-KKM property on locally G-convex uniform spaces. Amini-Harandi et al. [1] obtained fixed point theorems for Mönch type selfmultimaps in KKM class on $L\Gamma$-spaces. $L\Gamma$-spaces are abstract convex uniform spaces with local convexities. The concept of $L\Gamma$-space was introduced by Park [11] as a generalization of locally convex spaces, LG-spaces and other abstract locally convex structures.

The aim of this paper is to present new fixed point theorems for Mönch type multimaps on abstract convex uniform spaces. We obtain a fixed point theorem for Mönch type multimaps in Hausdorff KKM $L\Gamma$-spaces. We show that Mönch type multimaps in the 'better' admissible class defined on an $L\Gamma$-space have fixed point properties whenever their

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ranges are Klee approximable. And we obtain fixed point theorems on $\mathfrak{KC}$-maps whose ranges are Φ-sets. This result simplifies or generalizes the fixed point theorems in [1] and [2].

A *multimap* (or simply, a *map*) $F : X \multimap Y$ is a function from a set X into the power set of Y; that is, a function with the *values* $F(x) \subset Y$ for $x \in X$ and the *fibers* $F^{-}(y) := \{x \in X \,|\, y \in F(x)\}$ for $y \in Y$. For $A \subset X$, let $F(A) := \bigcup\{F(x) \,|\, x \in A\}$. Throughout this paper, we assume that multimaps have nonempty values otherwise explicitly stated or obvious from the context. The closure operation, the interior operation and graph of F are denoted by $\overline{\quad}$, Int$F$ and GrF, respectively.

Let $\langle X\rangle$ denote the set of all nonempty finite subsets of a set X.

The followings are due to Park [9, 11].

An *abstract convex space* $(X, D; \Gamma)$ consists of a topological space X, a non-empty set D, and a map $\Gamma : \langle D\rangle \multimap X$ with nonempty values $\Gamma_A := \Gamma(A)$ for $A \in \langle D\rangle$. For any nonempty $D' \subset D$, the Γ-*convex hull* of D' is denoted and defined by $\mathrm{co}_\Gamma D' := \bigcup\{\Gamma_A \,|\, A \in \langle D'\rangle\} \subset X$.

When $D \subset X$ in $(X, D; \Gamma)$, the space is denoted by $(X \supset D; \Gamma)$ and in case $X = D$, let $(X; \Gamma) := (X, X; \Gamma)$. When $(X \supset D; \Gamma)$, a subset X' of X is said to be Γ-*convex* if $\mathrm{co}_\Gamma(X' \cap D) \subset X'$. This means that $(X', D'; \Gamma')$ itself is an abstract convex space where $D' := X' \cap D$ and $\Gamma' : \langle D'\rangle \multimap X'$ a map defined by $\Gamma'_A := \Gamma_A \subset X'$ for $A \in \langle D'\rangle$.

An *abstract convex uniform space* $(X, D; \Gamma; \mathcal{U})$ is an abstract convex space with a basis $\mathcal{U}$ of a uniform structure of X. $A \subset X$ and $U \in \mathcal{U}$, the set $U[A]$ is defined to be $\{y \in X : (x, y) \in U \text{ for some } x \in A\}$. An abstract convex uniform space $(X \supset D; \Gamma; \mathcal{U})$ is called an $L\Gamma$-*space* if D is dense in X and $U[C]$ is Γ-convex for each $U \in \mathcal{U}$ whenever $C \subset X$ is Γ-convex.

A *generalized convex space* or a *G-convex space* $(X, D; \Gamma)$ consists of a topological space X such that for each $A \in \langle D\rangle$ with the cardinality $|A| = n + 1$, there exist a subset Γ_A of X and a continuous map $\phi_A : \Delta_n \to \Gamma_A$ such that $J \in \langle A\rangle$ implies $\phi_A(\Delta_J) \subset \Gamma_J$. Here, Δ_n is the standard n-simplex with vertices $\{e_0\}_{i=0}^{n}$, and Δ_J is the face of Δ_n corresponding to $J \in \langle A\rangle$; that is, if $A = \{a_0, a_1, \ldots, a_n\}$ and $J = \{a_{i_0}, a_{i_1}, \ldots, a_{i_k}\} \subset A$, then $\Delta_J = \mathrm{co}\{e_{i_0}, e_{i_1}, \ldots, e_{i_k}\}$. A subset S of X is called a *G-convex subset* of $(X \supset D; \Gamma)$ if for any $N \in \langle S\rangle$, we have $\Gamma_N \subset S$. For details on G-convex spaces, see [13, 14, 15].

A *G-convex uniform space* $(X, D; \Gamma; \mathcal{U})$ is a G-convex space with a basis $\mathcal{U}$ of a uniform structure of X. A G-convex uniform space $(X \supset D; \Gamma; \mathcal{U})$ is said to be an *LG-space* if the uniformity $\mathcal{U}$ has a base $\mathcal{B}$ such that for each $U \in \mathcal{B}$, $U[C]$ is Γ-convex for each $U \in \mathcal{U}$ whenever $C \subset X$ is Γ-convex. The examples of G-convex uniform space are given in [7].

Let $(X, D; \Gamma)$ be an abstract convex space and Z be a set. For a multimap $F : X \multimap Z$, if a multimap $G : D \multimap Z$ satisfies $F(\Gamma_A) \subset G(A)$ for all $A \in \langle D \rangle$, then G is called a *KKM map* with respect to F. A *KKM map* $G : D \multimap Z$ is a KKM map with respect to the identity map 1_X.

A multimap $F : X \multimap Z$ is called a $\mathfrak{K}$*-map* if, for a KKM map $G : D \multimap Z$ with respect to F, the family $\{G(x)\}_{x \in D}$ has the finite intersection property. The set $\mathfrak{K}(X, Z)$ is defined to be $\{F : X \multimap Z \mid F \text{ is a } \mathfrak{K}\text{-map}\}$. Similarly, a $\mathfrak{KC}$*-map* is defined for closed-valued maps G and a $\mathfrak{KO}$*-map* for open-valued maps G.

For an abstract convex space $(X, D; \Gamma)$, the *KKM principle* is the statement $1_X \in \mathfrak{KC}(X, X) \cap \mathfrak{KO}(X, X)$. An abstract convex space is called a *KKM space* if it satisfies the KKM principle. Known examples of KKM spaces are given in [10, 12] and the references therein. Note that a generalized convex space is also a KKM space.

Let $(X \supset D; \Gamma)$ be an abstract convex space, $A \subset X$ and put

$\Gamma\text{-co}A = \bigcap\{C \mid C \text{ is a } \Gamma\text{-convex subset of } X \text{ containing } A\}$, and

$\Gamma\text{-}\overline{\text{co}}A = \bigcap\{C \mid C \text{ is a closed } \Gamma\text{-convex subset of } X \text{ containing } A\}$.

Note that Γ-co$A$ and Γ-$\overline{\text{co}}A$ are the smallest Γ-convex set and the smallest closed Γ-convex set containing A, respectively. When $A \subset D$, $\text{co}_\Gamma A \subset \Gamma\text{-co}A$.

A subset S of a uniform space X is said to be *precompact* if, for any entourage V, there is an $N \in \langle X \rangle$ such that $S \subset V[N]$. For each $N \in \langle X \rangle$, Γ-co$N$ is called a *polytope* in X. An $L\Gamma$-space $(X \supset D; \Gamma; \mathcal{U})$ is called an *$L\Gamma$-space with precompact polytopes* if each polytope in X is precompact.

Note that Amini-Harandi et al [1] called an $L\Gamma$-space with precompact polytopes as an abstract convex uniform space.

The following lemma is in [5]:

LEMMA 1.1. *If $(X \supset D; \Gamma; \mathcal{U})$ is an $L\Gamma$-space with precompact polytopes and A is precompact, then Γ-co$A$ is precompact.*

2. Fixed point theorems for Mönch type maps

From now on we assume that every topological space is Hausdorff.

The following proposition is a crucial tool for Mönch type fixed point theorems:

Proposition 2.1. *Let* $\{(X \supset D; \Gamma; \mathcal{U})\}$ *be an* $L\Gamma$*-space with precompact polytopes. Suppose* $T : X \multimap X$ *is a map that satisfies the following properties:*

(1) T *maps compact sets into precompact sets;*
(2) *for any compact subset* A *of* X*, there exists a countable subset* B *of* A *with* $\overline{B} = A$*; and*
(3) *every subset* A *of* X *is compact, whenever* $A = \Gamma\text{-}\overline{\text{co}}(\{x_0\} \cup T(A))$ *for some* $x_0 \in X$ *and there exists a countable subset* C *of* A *with* $A = \overline{C}$.

Then there exists a nonempty compact Γ*-convex subset* K *of* X *such that* $T(K) \subset K$.

Proof. We are motivated by the proof of Theorem 2.4 in [1].

Choose $x_0 \in X$ and put $K_0 = \Gamma\text{-}\overline{\text{co}}(\{x_0\})$, $K_{n+1} = \Gamma\text{-}\overline{\text{co}}(\{x_0\} \cup T(K_n))$ for $n = 0, 1, 2, \cdots$ and $K = \bigcup_{n=0}^{\infty} K_n$. By induction, $K_0 \subseteq K_1 \subseteq \cdots \subseteq K_n \subseteq K_{n+1} \cdots$ and K is Γ-convex, since K_n is Γ-convex for $n = 0, 1, 2, \cdots$.

Furthermore we can show that $K = \Gamma\text{-}\overline{\text{co}}(\{x_0\} \cup T(K))$. For each n, $\Gamma\text{-}\overline{\text{co}}(\{x_0\} \cup T(K_n)) \subseteq \Gamma\text{-}\overline{\text{co}}(\{x_0\} \cup T(K))$, so $K = \bigcup_{n=0}^{\infty} \Gamma\text{-}\overline{\text{co}}(\{x_0\} \cup T(K_n)) \subseteq \Gamma\text{-}\overline{\text{co}}(\{x_0\} \cup T(K))$. On the other hand, K is a closed Γ-convex set which contains x_0 and $\bigcup_{n=0}^{\infty} T(K_n) = T(K)$, hence $\Gamma\text{-}\overline{\text{co}}(\{x_0\} \cup T(K)) \subseteq K$.

By (1) and Lemma 1.1, K_n is compact for $n = 0, 1, 2, \cdots$. Condition (2) implies that there exists a countable subset C_n of K_n with $\overline{C}_n = K_n$. Put $C = \bigcup_{n=0}^{\infty} C_n$, then $\overline{C} = K$, since $K = \bigcup_{n=0}^{\infty} K_n = \bigcup_{n=0}^{\infty} \overline{C_n} = \overline{\bigcup_{n=0}^{\infty} C_n} = \overline{C}$. Condition (3) implies that K is compact.

□

The following proposition is in [11]:

Proposition 2.2. *Let* $(X \supset D; \Gamma; \mathcal{U})$ *be a KKM* $L\Gamma$*-space and* $T : X \multimap X$ *be a compact upper semicontinuous map with closed* Γ*-convex values. Then* T *has a fixed point.*

Theorem 2.3. *Let* $(X \supset D; \Gamma; \mathcal{U})$ *be a KKM* $L\Gamma$*-space with precompact polytopes and* $T : X \multimap X$ *be a closed multimap with* Γ*-convex values that satisfies the following properties:*

(1) T *maps compact sets into precompact sets;*
(2) *for any compact subset* A *of* X*, there exists a countable subset* B *of* A *with* $\overline{B} = A$*; and*

(3) *every subset A of X is compact, whenever $A = \Gamma\text{-}\overline{\mathrm{co}}(\{x_0\} \cup T(A))$ for some $x_0 \in X$ and there exists a countable subset C of A with $A = \overline{C}$.*

Then T has a fixed point.

Proof. By Proposition 2.1, there exists a compact Γ-convex subset K of X such that $T(K) \subset K$. Since $T|_K$ is compact and closed, $T|_K$ is an upper semicontinuous map with closed Γ-convex values. By Proposition 2.2, $T|_K$ has a fixed point. □

Corollary 2.4. *Let $(X \supset D; \Gamma; \mathcal{U})$ be an LG-space with precompact polytopes and $T : X \multimap X$ be a closed multimap with Γ-convex values that satisfies the following properties:*

(1) *T maps compact sets into precompact sets;*
(2) *for any compact subset A of X, there exists a countable subset B of A with $\overline{B} = A$; and*
(3) *every subset A of X is compact, whenever $A = \Gamma\text{-}\overline{\mathrm{co}}(\{x_0\} \cup T(A))$ for some $x_0 \in X$ and there exists a countable subset C of A with $A = \overline{C}$.*

Then T has a fixed point.

Now, we follow the definitions in [11].

Let $(E, D; \Gamma)$ be an abstract convex space, X be a nonempty subset of E, and Y be a topological space. *The better admissible class* $\mathfrak{B}$ of maps from X into Y is defined as follows:

$F \in \mathfrak{B}(X, Y) \iff F : X \multimap Y$ is a map such that, for any $\Gamma_N \subset X$, where $N \in \langle D \rangle$ with the cardinality $|N| = n+1$, and for any continuous function $p : F(\Gamma_N) \to \Delta_n$, there exists a continuous function $\phi_N : \Delta_n \to \Gamma_N$ such that the composition $p \circ F|_{\Gamma_N} \circ \phi_N : \Delta_n \to \Delta_n$ has a fixed point.

Let $(E, D; \Gamma; \mathcal{U})$ be an abstract convex uniform space. A subset K of E is said to be *Klee approximable* if, for each entourage $U \in \mathcal{U}$, there exists a continuous function $h : K \to E$ satisfying

(1) $(x, h(x)) \in \mathcal{U}$ for all $x \in K$;

(2) $h(K) \subset \Gamma_N$ for some $N \in \langle D \rangle$; and

(3) there exist continuous functions $p : K \to \Delta_n$ and $\phi_N : \Delta_n \to \Gamma_N$ with $|N| = n + 1$ such that $h = \phi_N \circ p$.

The following proposition is a fixed point theorem for the class $\mathfrak{B}$ of multimaps in [11]:

Proposition 2.5. *Let* $(X \supset D; \Gamma; \mathcal{U})$ *be an abstract convex uniform space and* $T \in \mathfrak{B}(X, X)$ *be a closed map such that* $T(X)$ *is compact Klee approximable. Then* T *has a fixed point.*

Theorem 2.6. *Let* $(X \supset D; \Gamma; \mathcal{U})$ *be an* $L\Gamma$*-space with precompact polytopes and* $T \in \mathfrak{B}(X, X)$ *be a closed multimap that satisfies the following properties:*

(1) T *maps compact sets into precompact sets;*
(2) *for any compact subset* A *of* X*, there exists a countable subset* B *of* A *with* $\overline{B} = A$*;*
(3) *every subset* A *of* X *is compact, whenever* $A = \Gamma\text{-}\overline{\mathrm{co}}(\{x_0\} \cup T(A))$ *for some* $x_0 \in X$ *and there exists a countable subset* C *of* A *with* $A = \overline{C}$*; and*
(4) *for any compact* Γ*-convex subset* A *of* X*,* $T(A)$ *is Klee approximable.*

Then T *has a fixed point.*

Proof. By Proposition 2.1, there exists a compact Γ-convex subset K of X such that $T(K) \subset K$. Then $T|_K$ is closed and $T(K)$ is compact Klee approximable. $T \in \mathfrak{B}(X, X)$ implies $T|_K \in \mathfrak{B}(K, K)$. By Proposition 2.5, $T|_K$ has a fixed point. □

An abstract convex uniform space $(X; \Gamma; \mathcal{U})$ is called *admissible* iff every compact subset of X is Klee approximable. Therefore the following corollary holds:

Corollary 2.7. *Let* $(X \supset D; \Gamma; \mathcal{U})$ *be an admissible* $L\Gamma$*-space with precompact polytopes and* $T \in \mathfrak{B}(X, X)$ *be a closed multimap that satisfies the following properties:*

(1) T *maps compact sets into precompact sets;*
(2) *for any compact subset* A *of* X*, there exists a countable subset* B *of* A *with* $\overline{B} = A$*; and*
(3) *every subset* A *of* X *is compact, whenever* $A = \Gamma\text{-}\overline{\mathrm{co}}(\{x_0\} \cup T(A))$ *for some* $x_0 \in X$ *and there exists a countable subset* C *of* A *with* $A = \overline{C}$*.*

Then T *has a fixed point.*

For a given abstract convex space $(X \supset D; \Gamma)$ and a topological space Y, a map $H : Y \multimap X$ is called a Φ-*map* if there exists a map $G : Y \multimap X$ such that

(1) for each $y \in Y$, $\mathrm{co}_\Gamma G(y) \subset H(y)$; and
(2) $Y = \bigcup\{ \mathrm{Int} G^-(x) \mid x \in X\}$.

In an abstract convex uniform space $(X \supset D;\Gamma;\mathcal{U})$, a subset S of X is called a Φ*-set* if, for any entourage $U \in \mathcal{U}$, there exists a Φ-map $H : S \multimap X$ such that $\mathrm{Gr}H \subset U$.

Note that if a subset Y of X is a Φ-set, then any subset of A of Y is a Φ-set [1, Lemma 1.8].

The following propositions are in [9], [11]:

PROPOSITION 2.8. *Let $(X \supset D;\Gamma)$ be an abstract convex space, C be a Γ-convex subset of X and Z be a set. If $T \in \mathfrak{K}(X,Z)$, then $T|_C \in \mathfrak{K}(C,Z)$.*

PROPOSITION 2.9. *Let $(X \supset D;\Gamma;\mathcal{U})$ be an abstract convex uniform space, and $T \in \mathfrak{KC}(X,X)$ be a compact closed map. If $\overline{T(X)}$ is a Φ-set, then T has a fixed point.*

THEOREM 2.10. *Let $(X \supset D;\Gamma;\mathcal{U})$ be an $L\Gamma$-space with precompact polytopes, and $T \in \mathfrak{KC}(X,X)$ be a closed multimap that satisfies the following properties:*

1. *T maps compact sets into precompact sets;*
2. *for any compact subset A of X, there exists a countable subset B of A with $\overline{B} = A$;*
3. *every subset A of X is compact, whenever $A = \Gamma\text{-}\overline{\mathrm{co}}(\{x_0\} \cup T(A))$ for some $x_0 \in X$ and there exists a countable subset C of A with $A = \overline{C}$; and*
4. *$\overline{T(X)}$ is a Φ-set.*

Then T has a fixed point.

Proof. By Proposition 2.1, there exists a compact Γ-convex subset K of X such that $T(K) \subset K$. Then $T|_K$ is compact closed. Since $\overline{T(X)}$ is a Φ-set, so is $\overline{T(K)}$. By Proposition 2.8 and Proposition 2.9, $T|_K \in \mathfrak{KC}(K,K)$ and $T|_K$ has a fixed point. □

COROLLARY 2.11. *Let $(X \supset D;\Gamma;\mathcal{U})$ be an $L\Gamma$-space with precompact polytopes and X be a Φ-set. And let $T \in \mathfrak{KC}(X,X)$ be a closed multimap that satisfies the following properties:*

1. *T maps compact sets into precompact sets;*
2. *for any compact subset A of X, there exists a countable subset B of A with $\overline{B} = A$; and*
3. *every subset A of X is compact, whenever $A = \Gamma\text{-}\overline{\mathrm{co}}(\{x_0\} \cup T(A))$ for some $x_0 \in X$ and there exists a countable subset C of A with $A = \overline{C}$.*

Then T has a fixed point.

Note that Corollary 2.11 deletes a necessary condition of Theorem 2.4 in [1].

If every singleton of an LΓ-space $(X \supset D; \Gamma; \mathcal{U})$ is Γ-convex, then any subset of X is a Φ-set [11]. Therefore the following corollary holds:

COROLLARY 2.12. *Let* $(X \supset D; \Gamma; \mathcal{U})$ *be an* $L\Gamma$*-space with precompact polytopes and every singleton of* X *be* Γ*-convex. Let* $T \in \mathfrak{KC}(X, X)$ *be a closed multimap that satisfies the following properties:*

(1) T *maps compact sets into precompact sets;*
(2) *for any compact subset* A *of* X*, there exists a countable subset* B *of* A *with* $\overline{B} = A$*; and*
(3) *every subset* A *of* X *is compact, whenever* $A = \Gamma\text{-}\overline{\text{co}}(\{x_0\} \cup T(A))$ *for some* $x_0 \in X$ *and there exists a countable subset* C *of* A *with* $A = \overline{C}$*.*

Then T *has a fixed point.*

Let X be a nonempty set, $(Y; \Gamma)$ be an abstract convex space and Z be a topological space. If $S : X \multimap Y$, $T : Y \multimap Z$ and $F : X \multimap Z$ are three multimaps satisfying

$$T(\text{co}_\Gamma S(A)) \subset F(A) \qquad \text{for all } \ A \in \langle X \rangle,$$

then F is called an *S-KKM map* with respect to T. If for any S-KKM map F with respect to T, the family $\{\overline{F(x)}\}_{x \in X}$ has the finite intersection property, then T is said to have the *S-KKM property*. The class *S-KKM* (X, Y, Z) is defined to be the set $\{T : Y \multimap Z \,|\, T$ has the S-KKM property$\}$. If $X = Y$ and S is the identity map 1_X, then $S\text{-KKM}(X, Y, Z) = \mathfrak{KC}(X, Z)$.

It is shown that $s\text{-KKM}(X, Y, Z) \subset \mathfrak{KC}(Y, Z)$ for any surjective single valued function $s : X \to Y$ in [4], so Corollary 2.12 can be reformulated for $T \in s\text{-KKM}(Z, X, X)$.

An *LG*-space $(X; \Gamma; \mathcal{U})$ is said to be an *locally G-convex space* in [3] if $U[\{x\}]$ is Γ-convex for each $x \in X$ and $U \in \mathcal{U}$, and if $\Gamma\text{-co}A$ is precompact whenever A is precompact. So a locally G-convex space is an $L\Gamma$-space with precompact polytopes such that every singleton of X is Γ-convex. Therefore Corollary 2.12 generalizes and deletes an extra condition of Theorem 2.1 in [2].

Park [8] showed that $S\text{-KKM}(X, Y, Z)$ becomes $\mathfrak{KC}(Y, Z)$ by giving abstract convexity to the classical convex set Y.

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