

## DOUBLE PAIRWISE STRONGLY $(r, s)(u, v)$ -SEMIOPEN MAPPINGS

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**ABSTRACT.** We introduce the concepts of double pairwise strongly  $(r, s)(u, v)$ -semiopen mappings and double pairwise strongly  $(r, s)(u, v)$ -semiclosed mappings, and investigate some of their characteristic properties.

### 1. Introduction

As a generalization of fuzzy sets, the concept of intuitionistic fuzzy sets was introduced by Atanassov [1]. Çoker and his colleagues [4, 6, 7] introduced intuitionistic fuzzy topological spaces using intuitionistic fuzzy sets. Using the idea of degree of openness and degree of nonopenness, Çoker and Demirci [5] defined intuitionistic fuzzy topological spaces in Šostak's sense as a generalization of smooth fuzzy topological spaces and intuitionistic fuzzy topological spaces.

Kandil [8] introduced and studied the notion of fuzzy bitopological spaces as a natural generalization of fuzzy topological spaces.

We introduce the concepts of double pairwise strongly  $(r, s)(u, v)$ -semiopen mappings and double pairwise strongly  $(r, s)(u, v)$ -semiclosed mappings, and investigate some of their characteristic properties.

### 2. Preliminaries

Let  $I$  be the unit interval  $[0, 1]$  of the real line. A member  $\mu$  of  $I^X$  is called a fuzzy set of  $X$ . For any  $\mu \in I^X$ ,  $\mu^c$  denotes the complement  $1 - \mu$ . By  $\tilde{0}$  and  $\tilde{1}$  we denote constant maps on  $X$  with value 0 and

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1, respectively. All other notations are standard notations of fuzzy set theory.

Let  $X$  be a nonempty set. An *intuitionistic fuzzy set*  $A$  is an ordered pair

$$A = (\mu_A, \gamma_A)$$

where the functions  $\mu_A : X \rightarrow I$  and  $\gamma_A : X \rightarrow I$  denote the degree of membership and the degree of nonmembership, respectively, and  $\mu_A + \gamma_A \leq \tilde{1}$ . Obviously every fuzzy set  $\mu$  on  $X$  is an intuitionistic fuzzy set of the form  $(\mu, \tilde{1} - \mu)$ .

An *intuitionistic fuzzy topology* on  $X$  is a family  $T$  of intuitionistic fuzzy sets in  $X$  which satisfies the following properties:

- (1)  $0_\sim, 1_\sim \in T$ .
- (2) If  $A_1, A_2 \in T$ , then  $A_1 \cap A_2 \in T$ .
- (3) If  $A_i \in T$  for all  $i$ , then  $\bigcup A_i \in T$ .

The pair  $(X, T)$  is called an *intuitionistic fuzzy topological space*.

Let  $I(X)$  be a family of all intuitionistic fuzzy sets of  $X$  and let  $I \otimes I$  be the set of the pair  $(r, s)$  such that  $r, s \in I$  and  $r + s \leq 1$ .

DEFINITION 2.1. [5, 14] Let  $X$  be a nonempty set. An *intuitionistic fuzzy topology in Šostak's sense*  $\mathcal{T}^{\mu\gamma} = (\mathcal{T}^\mu, \mathcal{T}^\gamma)$  on  $X$  is a mapping  $\mathcal{T}^{\mu\gamma} : I(X) \rightarrow I \otimes I$  ( $\mathcal{T}^\mu, \mathcal{T}^\gamma : I(X) \rightarrow I$ ) which satisfies the following properties:

- (1)  $\mathcal{T}^\mu(0_\sim) = \mathcal{T}^\mu(1_\sim) = 1$  and  $\mathcal{T}^\gamma(0_\sim) = \mathcal{T}^\gamma(1_\sim) = 0$ .
- (2)  $\mathcal{T}^\mu(A \cap B) \geq \mathcal{T}^\mu(A) \wedge \mathcal{T}^\mu(B)$  and  $\mathcal{T}^\gamma(A \cap B) \leq \mathcal{T}^\gamma(A) \vee \mathcal{T}^\gamma(B)$ .
- (3)  $\mathcal{T}^\mu(\bigcup A_i) \geq \bigwedge \mathcal{T}^\mu(A_i)$  and  $\mathcal{T}^\gamma(\bigcup A_i) \leq \bigvee \mathcal{T}^\gamma(A_i)$ .

The  $(X, \mathcal{T}^{\mu\gamma})$  is said to be an *intuitionistic fuzzy topological space in Šostak's sense*. Also, we call  $\mathcal{T}^\mu(A)$  a *gradation of openness* of  $A$  and  $\mathcal{T}^\gamma(A)$  a *gradation of nonopenness* of  $A$ .

Let  $A$  be an intuitionistic fuzzy set in an intuitionistic fuzzy topological space in Šostak's sense  $(X, \mathcal{T}^{\mu\gamma})$  and  $(r, s) \in I \otimes I$ . Then  $A$  is said to be

- (1) a  $\mathcal{T}^{\mu\gamma}$ -fuzzy  $(r, s)$ -open set if  $\mathcal{T}^\mu(A) \geq r$  and  $\mathcal{T}^\gamma(A) \leq s$ ,
- (2) a  $\mathcal{T}^{\mu\gamma}$ -fuzzy  $(r, s)$ -closed set if  $\mathcal{T}^\mu(A^C) \geq r$  and  $\mathcal{T}^\gamma(A^C) \leq s$ .

Let  $(X, \mathcal{T}^{\mu\gamma})$  be an intuitionistic fuzzy topological space in Šostak's sense. For each  $(r, s) \in I \otimes I$  and for each  $A \in I(X)$ , the  $\mathcal{T}^{\mu\gamma}$ -fuzzy  $(r, s)$ -closure is defined by

$$\mathcal{T}^{\mu\gamma}\text{-cl}(A, (r, s)) = \bigcap \{B \in I(X) \mid A \subseteq B, B \text{ is } \mathcal{T}^{\mu\gamma}\text{-fuzzy } (r, s)\text{-closed}\}$$

and the  $\mathcal{T}^{\mu\gamma}$ -fuzzy  $(r, s)$ -interior is defined by

$$\mathcal{T}^{\mu\gamma}\text{-int}(A, (r, s)) = \bigcup \{B \in I(X) \mid A \supseteq B, B \text{ is } \mathcal{T}^{\mu\gamma}\text{-fuzzy } (r, s)\text{-open}\}.$$

A system  $(X, \mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$  consisting of a set  $X$  with two intuitionistic fuzzy topologies in Šostak's sense  $\mathcal{T}^{\mu\gamma}$  and  $\mathcal{U}^{\mu\gamma}$  on  $X$  is called a *double bitopological space*.

DEFINITION 2.2. [12] Let  $A$  be an intuitionistic fuzzy set of a double bitopological space  $(X, \mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$  and  $(r, s), (u, v) \in I \otimes I$ . Then  $A$  is said to be

- (1) a  $(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$ -double strongly  $(r, s)(u, v)$ -semiopen set if there is a  $\mathcal{T}^{\mu\gamma}$ -fuzzy  $(r, s)$ -open set  $B$  in  $X$  such that  $B \subseteq A \subseteq \mathcal{T}^{\mu\gamma}\text{-int}(\mathcal{U}^{\mu\gamma}\text{-cl}(B, (u, v)), (r, s))$ ,
- (2) a  $(\mathcal{U}^{\mu\gamma}, \mathcal{T}^{\mu\gamma})$ -double strongly  $(u, v)(r, s)$ -semiopen set if there is a  $\mathcal{U}^{\mu\gamma}$ -fuzzy  $(u, v)$ -open set  $B$  in  $X$  such that  $B \subseteq A \subseteq \mathcal{U}^{\mu\gamma}\text{-int}(\mathcal{T}^{\mu\gamma}\text{-cl}(B, (r, s)), (u, v))$ ,
- (3) a  $(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$ -double strongly  $(r, s)(u, v)$ -semiclosed set if there is a  $\mathcal{T}^{\mu\gamma}$ -fuzzy  $(r, s)$ -closed set  $B$  in  $X$  such that  $\mathcal{T}^{\mu\gamma}\text{-cl}(\mathcal{U}^{\mu\gamma}\text{-int}(B, (u, v)), (r, s)) \subseteq A \subseteq B$ ,
- (4) a  $(\mathcal{U}^{\mu\gamma}, \mathcal{T}^{\mu\gamma})$ -double strongly  $(u, v)(r, s)$ -semiclosed set if there is a  $\mathcal{U}^{\mu\gamma}$ -fuzzy  $(u, v)$ -closed set  $B$  in  $X$  such that  $\mathcal{U}^{\mu\gamma}\text{-cl}(\mathcal{T}^{\mu\gamma}\text{-int}(B, (r, s)), (u, v)) \subseteq A \subseteq B$ .

### 3. Double pairwise strongly $(r, s)(u, v)$ -semiopen mappings

DEFINITION 3.1. Let  $(X, \mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$  be a double bitopological space and  $(r, s), (u, v) \in I \otimes I$ . For each  $A \in I(X)$ , the  $(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$ -double strongly  $(r, s)(u, v)$ -semiclosure is defined by

$$(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})\text{-dsscl}(A, (r, s), (u, v)) = \bigcap \{B \in I(X) \mid B \supseteq A, \\ B \text{ is } (\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})\text{-double strongly } (r, s)(u, v)\text{-semiclosed}\}$$

and the  $(\mathcal{U}^{\mu\gamma}, \mathcal{T}^{\mu\gamma})$ -double strongly  $(u, v)(r, s)$ -semiclosure is defined by

$$(\mathcal{U}^{\mu\gamma}, \mathcal{T}^{\mu\gamma})\text{-dsscl}(A, (u, v), (r, s)) = \bigcap \{B \in I(X) \mid B \supseteq A, \\ B \text{ is } (\mathcal{U}^{\mu\gamma}, \mathcal{T}^{\mu\gamma})\text{-double strongly } (u, v)(r, s)\text{-semiclosed}\}.$$

DEFINITION 3.2. Let  $(X, \mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$  be a double bitopological space and  $(r, s), (u, v) \in I \otimes I$ . For each  $A \in I(X)$ , the  $(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$ -double

*strongly*  $(r, s)(u, v)$ -semiinterior is defined by

$$(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})\text{-dssint}(A, (r, s), (u, v)) = \bigcup \{B \in I(X) \mid B \subseteq A,$$

$$B \text{ is } (\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})\text{-double strongly } (r, s)(u, v)\text{-semiopen}\}$$

and the  $(\mathcal{U}^{\mu\gamma}, \mathcal{T}^{\mu\gamma})$ -double strongly  $(u, v)(r, s)$ -semiinterior is defined by

$$(\mathcal{U}^{\mu\gamma}, \mathcal{T}^{\mu\gamma})\text{-dssint}(A, (u, v), (r, s)) = \bigcup \{B \in I(X) \mid B \subseteq A,$$

$$B \text{ is } (\mathcal{U}^{\mu\gamma}, \mathcal{T}^{\mu\gamma})\text{-double strongly } (u, v)(r, s)\text{-semiopen}\}.$$

DEFINITION 3.3. Let  $f : (X, \mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma}) \rightarrow (Y, \mathcal{V}^{\mu\gamma}, \mathcal{W}^{\mu\gamma})$  be a mapping from a double bitopological space  $X$  to a double bitopological space  $Y$  and  $(r, s)(u, v) \in I \otimes I$ . Then  $f$  is called

- (1) a *double pairwise strongly*  $(r, s)(u, v)$ -semiopen mapping if  $f(A)$  is a  $(\mathcal{V}^{\mu\gamma}, \mathcal{W}^{\mu\gamma})$ -double strongly  $(r, s)(u, v)$ -semiopen set of  $Y$  for each  $\mathcal{T}^{\mu\gamma}$ -fuzzy  $(r, s)$ -open set  $A$  of  $X$  and  $f(B)$  is a  $(\mathcal{W}^{\mu\gamma}, \mathcal{V}^{\mu\gamma})$ -double strongly  $(u, v)(r, s)$ -semiopen set of  $Y$  for each  $\mathcal{U}^{\mu\gamma}$ -fuzzy  $(u, v)$ -open set  $B$  of  $X$ ,
- (2) a *double pairwise strongly*  $(r, s)(u, v)$ -semiclosed mapping if  $f(A)$  is a  $(\mathcal{V}^{\mu\gamma}, \mathcal{W}^{\mu\gamma})$ -double strongly  $(r, s)(u, v)$ -semiclosed set of  $Y$  for each  $\mathcal{T}^{\mu\gamma}$ -fuzzy  $(r, s)$ -closed set  $A$  of  $X$  and  $f(B)$  is a  $(\mathcal{W}^{\mu\gamma}, \mathcal{V}^{\mu\gamma})$ -double strongly  $(u, v)(r, s)$ -semiclosed set of  $Y$  for each  $\mathcal{U}^{\mu\gamma}$ -fuzzy  $(u, v)$ -closed set  $B$  of  $X$ .

THEOREM 3.4. Let  $f : (X, \mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma}) \rightarrow (Y, \mathcal{V}^{\mu\gamma}, \mathcal{W}^{\mu\gamma})$  be a mapping and  $(r, s), (u, v) \in I \otimes I$ . Then the following statements are equivalent:

- (1)  $f$  is a double pairwise strongly  $(r, s)(u, v)$ -semiopen mapping.
- (2) For each intuitionistic fuzzy set  $A$  of  $X$ ,

$$f(\mathcal{T}^{\mu\gamma}\text{-int}(A, (r, s))) \subseteq (\mathcal{V}^{\mu\gamma}, \mathcal{W}^{\mu\gamma})\text{-dssint}(f(A), (r, s), (u, v))$$

and

$$f(\mathcal{U}^{\mu\gamma}\text{-int}(A, (u, v))) \subseteq (\mathcal{W}^{\mu\gamma}, \mathcal{V}^{\mu\gamma})\text{-dssint}(f(A), (u, v), (r, s)).$$

- (3) For each intuitionistic fuzzy set  $C$  of  $Y$ ,

$$\mathcal{T}^{\mu\gamma}\text{-int}(f^{-1}(C), (r, s)) \subseteq f^{-1}((\mathcal{V}^{\mu\gamma}, \mathcal{W}^{\mu\gamma})\text{-dssint}(C, (r, s), (u, v)))$$

and

$$\mathcal{U}^{\mu\gamma}\text{-int}(f^{-1}(C), (u, v)) \subseteq f^{-1}((\mathcal{W}^{\mu\gamma}, \mathcal{V}^{\mu\gamma})\text{-dssint}(C, (u, v), (r, s))).$$

*Proof.* (1)  $\Rightarrow$  (2) Let  $A$  be any intuitionistic fuzzy set of  $X$ . Clearly  $\mathcal{T}^{\mu\gamma}\text{-int}(A, (r, s))$  is a  $\mathcal{T}^{\mu\gamma}$ -fuzzy  $(r, s)$ -open set and  $\mathcal{U}^{\mu\gamma}\text{-int}(A, (u, v))$  is a  $\mathcal{U}^{\mu\gamma}$ -fuzzy  $(u, v)$ -open set of  $X$ . Since  $f$  is a double pairwise strongly  $(r, s)(u, v)$ -semiopen mapping,  $f(\mathcal{T}^{\mu\gamma}\text{-int}(A, (r, s)))$  is a  $(\mathcal{V}^{\mu\gamma}, \mathcal{W}^{\mu\gamma})$ -double

strongly  $(r, s)(u, v)$ -semiopen set and  $f(\mathcal{V}^{\mu\gamma}\text{-int}(A, (u, v)))$  is a  $(\mathcal{W}^{\mu\gamma}, \mathcal{V}^{\mu\gamma})$ -double strongly  $(u, v)(r, s)$ -semiopen set of  $Y$ . Thus

$$\begin{aligned} & f(\mathcal{T}^{\mu\gamma}\text{-int}(A, (r, s))) \\ &= (\mathcal{V}^{\mu\gamma}, \mathcal{W}^{\mu\gamma})\text{-dssint}(f(\mathcal{T}^{\mu\gamma}\text{-int}(A, (r, s))), (r, s), (u, v)) \\ &\subseteq (\mathcal{V}^{\mu\gamma}, \mathcal{W}^{\mu\gamma})\text{-dssint}(f(A), (r, s), (u, v)) \end{aligned}$$

and

$$\begin{aligned} & f(\mathcal{U}^{\mu\gamma}\text{-int}(A, (u, v))) \\ &= (\mathcal{W}^{\mu\gamma}, \mathcal{V}^{\mu\gamma})\text{-dssint}(f(\mathcal{U}^{\mu\gamma}\text{-int}(A, (u, v))), (u, v), (r, s)) \\ &\subseteq (\mathcal{W}^{\mu\gamma}, \mathcal{V}^{\mu\gamma})\text{-dssint}(f(A), (u, v), (r, s)). \end{aligned}$$

(2)  $\Rightarrow$  (3) Let  $C$  be any intuitionistic fuzzy set of  $Y$ . Then  $f^{-1}(C)$  is an intuitionistic fuzzy set of  $X$ . By (2),

$$\begin{aligned} f(\mathcal{T}^{\mu\gamma}\text{-int}(f^{-1}(C), (r, s))) &\subseteq (\mathcal{V}^{\mu\gamma}, \mathcal{W}^{\mu\gamma})\text{-dssint}(ff^{-1}(C), (r, s), (u, v)) \\ &\subseteq (\mathcal{V}^{\mu\gamma}, \mathcal{W}^{\mu\gamma})\text{-dssint}(C, (r, s), (u, v)) \end{aligned}$$

and

$$\begin{aligned} f(\mathcal{U}^{\mu\gamma}\text{-int}(f^{-1}(C), (u, v))) &\subseteq (\mathcal{W}^{\mu\gamma}, \mathcal{V}^{\mu\gamma})\text{-dssint}(ff^{-1}(C), (u, v), (r, s)) \\ &\subseteq (\mathcal{W}^{\mu\gamma}, \mathcal{V}^{\mu\gamma})\text{-dssint}(C, (u, v), (r, s)). \end{aligned}$$

Then we have

$$\begin{aligned} \mathcal{T}^{\mu\gamma}\text{-int}(f^{-1}(C), (r, s)) &\subseteq f^{-1}f(\mathcal{T}^{\mu\gamma}\text{-int}(f^{-1}(C), (r, s))) \\ &\subseteq f^{-1}((\mathcal{V}^{\mu\gamma}, \mathcal{W}^{\mu\gamma})\text{-dssint}(C, (r, s), (u, v))) \end{aligned}$$

and

$$\begin{aligned} \mathcal{U}^{\mu\gamma}\text{-int}(f^{-1}(C), (u, v)) &\subseteq f^{-1}f(\mathcal{U}^{\mu\gamma}\text{-int}(f^{-1}(C), (u, v))) \\ &\subseteq f^{-1}((\mathcal{W}^{\mu\gamma}, \mathcal{V}^{\mu\gamma})\text{-dssint}(C, (u, v), (r, s))). \end{aligned}$$

(3)  $\Rightarrow$  (1) Let  $A$  be any  $\mathcal{T}^{\mu\gamma}$ -fuzzy  $(r, s)$ -open set and  $B$  any  $\mathcal{U}^{\mu\gamma}$ -fuzzy  $(u, v)$ -open set of  $X$ . Then  $A = \mathcal{T}^{\mu\gamma}\text{-int}(A, (r, s))$  and  $B = \mathcal{U}^{\mu\gamma}\text{-int}(B, (u, v))$ . By (3),

$$\begin{aligned} A = \mathcal{T}^{\mu\gamma}\text{-int}(A, (r, s)) &\subseteq \mathcal{T}^{\mu\gamma}\text{-int}(f^{-1}f(A), (r, s)) \\ &\subseteq f^{-1}((\mathcal{V}^{\mu\gamma}, \mathcal{W}^{\mu\gamma})\text{-dssint}(f(A), (r, s), (u, v))) \end{aligned}$$

and

$$\begin{aligned} B = \mathcal{U}^{\mu\gamma}\text{-int}(B, (u, v)) &\subseteq \mathcal{U}^{\mu\gamma}\text{-int}(f^{-1}f(B), (u, v)) \\ &\subseteq f^{-1}((\mathcal{W}^{\mu\gamma}, \mathcal{V}^{\mu\gamma})\text{-dssint}(f(B), (u, v), (r, s))). \end{aligned}$$

Thus

$$\begin{aligned} f(A) &\subseteq f f^{-1}((\mathcal{V}^{\mu\gamma}, \mathcal{W}^{\mu\gamma})\text{-dssint}(f(A), (r, s), (u, v))) \\ &\subseteq (\mathcal{V}^{\mu\gamma}, \mathcal{W}^{\mu\gamma})\text{-dssint}(f(A), (r, s), (u, v)) \\ &\subseteq f(A) \end{aligned}$$

and

$$\begin{aligned} f(B) &\subseteq f f^{-1}((\mathcal{W}^{\mu\gamma}, \mathcal{V}^{\mu\gamma})\text{-dssint}(f(B), (u, v), (r, s))) \\ &\subseteq (\mathcal{W}^{\mu\gamma}, \mathcal{V}^{\mu\gamma})\text{-dssint}(f(B), (u, v), (r, s)) \\ &\subseteq f(B). \end{aligned}$$

So  $f(A) = (\mathcal{V}^{\mu\gamma}, \mathcal{W}^{\mu\gamma})\text{-dssint}(f(A), (r, s), (u, v))$  and  $f(B) = (\mathcal{W}^{\mu\gamma}, \mathcal{V}^{\mu\gamma})\text{-dssint}(f(B), (u, v), (r, s))$ . Hence  $f(A)$  is a  $(\mathcal{V}^{\mu\gamma}, \mathcal{W}^{\mu\gamma})$ -double strongly  $(r, s)(u, v)$ -semiopen set and  $f(B)$  is a  $(\mathcal{W}^{\mu\gamma}, \mathcal{V}^{\mu\gamma})$ -double strongly  $(u, v)(r, s)$ -semiopen set of  $Y$ . Therefore  $f$  is a double pairwise strongly  $(r, s)(u, v)$ -semiopen mapping.  $\square$

**THEOREM 3.5.** Let  $f : (X, \mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma}) \rightarrow (Y, \mathcal{V}^{\mu\gamma}, \mathcal{W}^{\mu\gamma})$  be a mapping and  $(r, s), (u, v) \in I \otimes I$ . Then the following statements are equivalent:

- (1)  $f$  is a double pairwise strongly  $(r, s)(u, v)$ -semiclosed mapping.
- (2) For each intuitionistic fuzzy set  $A$  of  $X$ ,

$$(\mathcal{V}^{\mu\gamma}, \mathcal{W}^{\mu\gamma})\text{-dsscl}(f(A), (r, s), (u, v)) \subseteq f(\mathcal{T}^{\mu\gamma}\text{-cl}(A, (r, s)))$$

and

$$(\mathcal{W}^{\mu\gamma}, \mathcal{V}^{\mu\gamma})\text{-dsscl}(f(A), (u, v), (r, s)) \subseteq f(\mathcal{U}^{\mu\gamma}\text{-cl}(A, (u, v))).$$

*Proof.* (1)  $\Rightarrow$  (2) Let  $A$  be any intuitionistic fuzzy set of  $X$ . Clearly  $\mathcal{T}^{\mu\gamma}\text{-cl}(A, (r, s))$  is a  $\mathcal{T}^{\mu\gamma}$ -fuzzy  $(r, s)$ -closed set and  $\mathcal{U}^{\mu\gamma}\text{-cl}(A, (u, v))$  is a  $\mathcal{U}^{\mu\gamma}$ -fuzzy  $(u, v)$ -closed set of  $X$ . Since  $f$  is a double pairwise strongly  $(r, s)(u, v)$ -semiclosed mapping,  $f(\mathcal{T}^{\mu\gamma}\text{-cl}(A, (r, s)))$  is a  $(\mathcal{V}^{\mu\gamma}, \mathcal{W}^{\mu\gamma})$ -double strongly  $(r, s)(u, v)$ -semiclosed set and  $f(\mathcal{U}^{\mu\gamma}\text{-cl}(A, (u, v)))$  is a  $(\mathcal{W}^{\mu\gamma}, \mathcal{V}^{\mu\gamma})$ -double strongly  $(u, v)(r, s)$ -semiclosed set of  $Y$ . Thus

$$\begin{aligned} f(\mathcal{T}^{\mu\gamma}\text{-cl}(A, (r, s))) &= (\mathcal{V}^{\mu\gamma}, \mathcal{W}^{\mu\gamma})\text{-dsscl}(f(\mathcal{T}^{\mu\gamma}\text{-cl}(A, (r, s))), (r, s), (u, v)) \\ &\supseteq (\mathcal{V}^{\mu\gamma}, \mathcal{W}^{\mu\gamma})\text{-dsscl}(f(A), (r, s), (u, v)) \end{aligned}$$

and

$$\begin{aligned} f(\mathcal{U}^{\mu\gamma}\text{-cl}(A, (u, v))) &= (\mathcal{W}^{\mu\gamma}, \mathcal{V}^{\mu\gamma})\text{-dsscl}(f(\mathcal{U}^{\mu\gamma}\text{-cl}(A, (u, v))), (u, v), (r, s)) \\ &\supseteq (\mathcal{W}^{\mu\gamma}, \mathcal{V}^{\mu\gamma})\text{-dsscl}(f(A), (u, v), (r, s)). \end{aligned}$$

(2)  $\Rightarrow$  (1) Let  $A$  be any  $\mathcal{T}^{\mu\gamma}$ -fuzzy  $(r, s)$ -closed set and  $B$  any  $\mathcal{U}^{\mu\gamma}$ -fuzzy  $(u, v)$ -closed set of  $X$ . Then  $A = \mathcal{T}^{\mu\gamma}\text{-cl}(A, (r, s))$  and  $B =$

$\mathcal{U}^{\mu\gamma}\text{-cl}(B, (u, v))$ . By (2),

$$\begin{aligned} & (\mathcal{V}^{\mu\gamma}, \mathcal{W}^{\mu\gamma})\text{-dsscl}(f(A), (r, s), (u, v)) \\ & \subseteq f(\mathcal{T}^{\mu\gamma}\text{-cl}(A, (r, s))) \\ & = f(A) \\ & \subseteq (\mathcal{V}^{\mu\gamma}, \mathcal{W}^{\mu\gamma})\text{-dsscl}(f(A), (r, s), (u, v)) \end{aligned}$$

and

$$\begin{aligned} & (\mathcal{W}^{\mu\gamma}, \mathcal{V}^{\mu\gamma})\text{-dsscl}(f(B), (u, v), (r, s)) \\ & \subseteq f(\mathcal{U}^{\mu\gamma}\text{-cl}(B, (u, v))) \\ & = f(B) \\ & \subseteq (\mathcal{W}^{\mu\gamma}, \mathcal{V}^{\mu\gamma})\text{-dsscl}(f(B), (u, v), (r, s)). \end{aligned}$$

So  $f(A) = (\mathcal{V}^{\mu\gamma}, \mathcal{W}^{\mu\gamma})\text{-dsscl}(f(A), (r, s), (u, v))$  and  $f(B) = (\mathcal{W}^{\mu\gamma}, \mathcal{V}^{\mu\gamma})\text{-dsscl}(f(B), (u, v), (r, s))$ . Hence  $f(A)$  is a  $(\mathcal{V}^{\mu\gamma}, \mathcal{W}^{\mu\gamma})$ -double strongly  $(r, s)(u, v)$ -semiclosed set and  $f(B)$  is a  $(\mathcal{W}^{\mu\gamma}, \mathcal{V}^{\mu\gamma})$ -double strongly  $(u, v)(r, s)$ -semiclosed set of  $Y$ . Therefore  $f$  is a double pairwise strongly  $(r, s)(u, v)$ -semiclosed mapping.  $\square$

**THEOREM 3.6.** Let  $f : (X, \mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma}) \rightarrow (Y, \mathcal{V}^{\mu\gamma}, \mathcal{W}^{\mu\gamma})$  be a bijection and  $(r, s), (u, v) \in I \otimes I$ . Then  $f$  is a double pairwise strongly  $(r, s)(u, v)$ -semiclosed mapping if and only if for each intuitionistic fuzzy set  $C$  of  $Y$ ,

$$f^{-1}((\mathcal{V}^{\mu\gamma}, \mathcal{W}^{\mu\gamma})\text{-dsscl}(C, (r, s), (u, v))) \subseteq \mathcal{T}^{\mu\gamma}\text{-cl}(f^{-1}(C), (r, s))$$

and

$$f^{-1}((\mathcal{W}^{\mu\gamma}, \mathcal{V}^{\mu\gamma})\text{-dsscl}(C, (u, v), (r, s))) \subseteq \mathcal{U}^{\mu\gamma}\text{-cl}(f^{-1}(C), (u, v)).$$

*Proof.* Let  $C$  be any intuitionistic fuzzy set of  $Y$ . Then  $f^{-1}(C)$  is an intuitionistic fuzzy set of  $X$ . Since  $f$  is onto,

$$\begin{aligned} & (\mathcal{V}^{\mu\gamma}, \mathcal{W}^{\mu\gamma})\text{-dsscl}(C, (r, s), (u, v)) \\ & = (\mathcal{V}^{\mu\gamma}, \mathcal{W}^{\mu\gamma})\text{-dsscl}(ff^{-1}(C), (r, s), (u, v)) \\ & \subseteq f(\mathcal{T}^{\mu\gamma}\text{-cl}(f^{-1}(C), (r, s))) \end{aligned}$$

and

$$\begin{aligned} & (\mathcal{W}^{\mu\gamma}, \mathcal{V}^{\mu\gamma})\text{-dsscl}(C, (u, v), (r, s)) \\ & = (\mathcal{W}^{\mu\gamma}, \mathcal{V}^{\mu\gamma})\text{-dsscl}(ff^{-1}(C), (u, v), (r, s)) \\ & \subseteq f(\mathcal{U}^{\mu\gamma}\text{-cl}(f^{-1}(C), (u, v))). \end{aligned}$$

Since  $f$  is one-to-one, we have

$$\begin{aligned} & f^{-1}((\mathcal{V}^{\mu\gamma}, \mathcal{W}^{\mu\gamma})\text{-dsscl}(C, (r, s), (u, v))) \\ & \subseteq f^{-1}f(\mathcal{T}^{\mu\gamma}\text{-cl}(f^{-1}(C), (r, s))) \\ & = \mathcal{T}^{\mu\gamma}\text{-cl}(f^{-1}(C), (r, s)) \end{aligned}$$

and

$$\begin{aligned} & f^{-1}((\mathcal{W}^{\mu\gamma}, \mathcal{V}^{\mu\gamma})\text{-dsscl}(C, (u, v), (r, s))) \\ & \subseteq f^{-1}f(\mathcal{U}^{\mu\gamma}\text{-cl}(f^{-1}(C), (u, v))) \\ & = \mathcal{U}^{\mu\gamma}\text{-cl}(f^{-1}(C), (u, v)). \end{aligned}$$

Conversely, let  $A$  be any  $\mathcal{T}^{\mu\gamma}$ -fuzzy  $(r, s)$ -closed set and  $B$  any  $\mathcal{U}^{\mu\gamma}$ -fuzzy  $(u, v)$ -closed set of  $X$ . Then  $A = \mathcal{T}^{\mu\gamma}\text{-cl}(A, (r, s))$  and  $B = \mathcal{U}^{\mu\gamma}\text{-cl}(B, (u, v))$ . Since  $f$  is one-to-one,

$$\begin{aligned} & f^{-1}((\mathcal{V}^{\mu\gamma}, \mathcal{W}^{\mu\gamma})\text{-dsscl}(f(A), (r, s), (u, v))) \\ & \subseteq \mathcal{T}^{\mu\gamma}\text{-cl}(f^{-1}f(A), (r, s)) \\ & = \mathcal{T}^{\mu\gamma}\text{-cl}(A, (r, s)) \\ & = A \end{aligned}$$

and

$$\begin{aligned} & f^{-1}((\mathcal{W}^{\mu\gamma}, \mathcal{V}^{\mu\gamma})\text{-dsscl}(f(B), (u, v), (r, s))) \\ & \subseteq \mathcal{U}^{\mu\gamma}\text{-cl}(f^{-1}f(B), (u, v)) \\ & = \mathcal{U}^{\mu\gamma}\text{-cl}(B, (u, v)) \\ & = B. \end{aligned}$$

Since  $f$  is onto, we have

$$\begin{aligned} & (\mathcal{V}^{\mu\gamma}, \mathcal{W}^{\mu\gamma})\text{-dsscl}(f(A), (r, s), (u, v)) \\ & = f f^{-1}((\mathcal{V}^{\mu\gamma}, \mathcal{W}^{\mu\gamma})\text{-dsscl}(f(A), (r, s), (u, v))) \\ & \subseteq f(A) \\ & \subseteq (\mathcal{V}^{\mu\gamma}, \mathcal{W}^{\mu\gamma})\text{-dsscl}(f(A), (r, s), (u, v)) \end{aligned}$$

and

$$\begin{aligned} & (\mathcal{W}^{\mu\gamma}, \mathcal{V}^{\mu\gamma})\text{-dsscl}(f(B), (u, v), (r, s)) \\ & = f f^{-1}((\mathcal{W}^{\mu\gamma}, \mathcal{V}^{\mu\gamma})\text{-dsscl}(f(B), (u, v), (r, s))) \\ & \subseteq f(B) \\ & \subseteq (\mathcal{W}^{\mu\gamma}, \mathcal{V}^{\mu\gamma})\text{-dsscl}(f(B), (u, v), (r, s)). \end{aligned}$$

Thus  $f(A) = (\mathcal{V}^{\mu\gamma}, \mathcal{W}^{\mu\gamma})$ -dsscl( $f(A), (r, s), (u, v)$ ) and  $f(B) = (\mathcal{W}^{\mu\gamma}, \mathcal{V}^{\mu\gamma})$ -dsscl( $f(B), (u, v), (r, s)$ ). Hence  $f(A)$  is a  $(\mathcal{V}^{\mu\gamma}, \mathcal{W}^{\mu\gamma})$ -double strongly  $(r, s)(u, v)$ -semiclosed set and  $f(B)$  is a  $(\mathcal{W}^{\mu\gamma}, \mathcal{V}^{\mu\gamma})$ -double strongly  $(u, v)(r, s)$ -semiclosed set of  $Y$ . Therefore  $f$  is a double pairwise strongly  $(r, s)(u, v)$ -semiclosed mapping.  $\square$

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