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DOUBLE PAIRWISE $(r, s)(u, v)$-PRECONTINUOUS MAPPINGS

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Abstract. We introduce the concepts of $(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$-double $(r, s)(u, v)$-preclosures and $(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$-double $(r, s)(u, v)$-preinteriors. Using the notions, we investigate some of characteristic properties of double pairwise $(r, s)(u, v)$-precontinuous mappings.

1. Introduction

Chang [2] defined fuzzy topological spaces. These spaces and its generalizations are later studied by several authors, one of which, developed by Šostak [12], used the idea of degree of openness. This type of generalization of fuzzy topological spaces was later rephrased by Chattopadhyay, Hazra, and Samanta [3], and by Ramadan [11].

As a generalization of fuzzy sets, the concept of intuitionistic fuzzy sets was introduced by Atanassov [1]. Çoker and his colleagues [4, 6, 7] introduced intuitionistic fuzzy topological spaces using intuitionistic fuzzy sets. Using the idea of degree of openness and degree of nonopenness, Çoker and Demirci [5] defined intuitionistic fuzzy topological spaces in Šostak's sense as a generalization of smooth fuzzy topological spaces and intuitionistic fuzzy topological spaces.

Kandil [8] introduced and studied the notion of fuzzy bitopological spaces as a natural generalization of fuzzy topological spaces.

In this paper, we introduce the concepts of $(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$-double $(r, s)(u, v)$-preclosures and $(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$-double $(r, s)(u, v)$-preinteriors. Using the notions, we investigate some of characteristic properties of double pairwise $(r, s)(u, v)$-precontinuous mappings.

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2. Preliminaries

Let I be the unit interval $[0,1]$ of the real line. A member μ of I^X is called a fuzzy set of X. For any $\mu \in I^X$, μ^c denotes the complement $1-\mu$. By $\tilde{0}$ and $\tilde{1}$ we denote constant maps on X with value 0 and 1, respectively. All other notations are standard notations of fuzzy set theory. Let X be a nonempty set. An *intuitionistic fuzzy set* A is an ordered pair

$$A = (\mu_A, \gamma_A)$$

where the functions $\mu_A : X \to I$ and $\gamma_A : X \to I$ denote the degree of membership and the degree of nonmembership, respectively, and $\mu_A + \gamma_A \leq \tilde{1}$.

Obviously every fuzzy set μ on X is an intuitionistic fuzzy set of the form $(\mu, \tilde{1} - \mu)$.

Definition 2.1. [1] Let $A = (\mu_A, \gamma_A)$ and $B = (\mu_B, \gamma_B)$ be intuitionistic fuzzy sets on X. Then

(1) $A \subseteq B$ iff $\mu_A \leq \mu_B$ and $\gamma_A \geq \gamma_B$.
(2) $A = B$ iff $A \subseteq B$ and $B \subseteq A$.
(3) $A^c = (\gamma_A, \mu_A)$.
(4) $A \cap B = (\mu_A \wedge \mu_B, \gamma_A \vee \gamma_B)$.
(5) $A \cup B = (\mu_A \vee \mu_B, \gamma_A \wedge \gamma_B)$.
(6) $0_\sim = (\tilde{0}, \tilde{1})$ and $1_\sim = (\tilde{1}, \tilde{0})$.

Let f be a mapping from a set X to a set Y. Let $A = (\mu_A, \gamma_A)$ be an intuitionistic fuzzy set of X and $B = (\mu_B, \gamma_B)$ an intuitionistic fuzzy set of Y. Then:

(1) The image of A under f, denoted by $f(A)$, is an intuitionistic fuzzy set in Y defined by
$$f(A) = (f(\mu_A), \tilde{1} - f(\tilde{1} - \gamma_A)).$$
(2) The inverse image of B under f, denoted by $f^{-1}(B)$, is an intuitionistic fuzzy set in X defined by
$$f^{-1}(B) = (f^{-1}(\mu_B), f^{-1}(\gamma_B)).$$

An *intuitionistic fuzzy topology* on X is a family T of intuitionistic fuzzy sets in X which satisfies the following properties:

(1) $0_\sim, 1_\sim \in T$.
(2) If $A_1, A_2 \in T$, then $A_1 \cap A_2 \in T$.
(3) If $A_i \in T$ for all i, then $\bigcup A_i \in T$.

The pair (X, T) is called an *intuitionistic fuzzy topological space.*

Let $I(X)$ be a family of all intuitionistic fuzzy sets of X and let $I \otimes I$ be the set of the pair (r,s) such that $r,s \in I$ and $r+s \leq 1$.

Definition 2.2. [12] Let X be a nonempty set. An *intuitionistic fuzzy topology in Šostak's sense* $\mathcal{T}^{\mu\gamma} = (\mathcal{T}^{\mu}, \mathcal{T}^{\gamma})$ on X is a mapping $\mathcal{T}^{\mu\gamma} : I(X) \to I \otimes I(\mathcal{T}^{\mu}, \mathcal{T}^{\gamma} : I(X) \to I)$ which satisfies the following properties:

(1) $\mathcal{T}^{\mu}(0_{\sim}) = \mathcal{T}^{\mu}(1_{\sim}) = 1$ and $\mathcal{T}^{\gamma}(0_{\sim}) = \mathcal{T}^{\gamma}(1_{\sim}) = 0$.
(2) $\mathcal{T}^{\mu}(A \cap B) \geq \mathcal{T}^{\mu}(A) \wedge \mathcal{T}^{\mu}(B)$ and $\mathcal{T}^{\gamma}(A \cap B) \leq \mathcal{T}^{\gamma}(A) \vee \mathcal{T}^{\gamma}(B)$.
(3) $\mathcal{T}^{\mu}(\bigcup A_i) \geq \bigwedge \mathcal{T}^{\mu}(A_i)$ and $\mathcal{T}^{\gamma}(\bigcup A_i) \leq \bigvee \mathcal{T}^{\gamma}(A_i)$.

The $(X, \mathcal{T}^{\mu\gamma}) = (X, \mathcal{T}^{\mu}, \mathcal{T}^{\gamma})$ is said to be an *intuitionistic fuzzy topological space in Šostak's sense.* Also, we call $\mathcal{T}^{\mu}(A)$ a *gradation of openness* of A and $\mathcal{T}^{\gamma}(A)$ a *gradation of nonopenness* of A.

Let A be an intuitionistic fuzzy set in an intuitionistic fuzzy topological space in Šostak's sense $(X, \mathcal{T}^{\mu}, \mathcal{T}^{\gamma})$ and $(r,s) \in I \otimes I$. Then A is said to be

(1) a $\mathcal{T}^{\mu\gamma}$-*fuzzy* (r,s)-*open* set if $\mathcal{T}^{\mu}(A) \geq r$ and $\mathcal{T}^{\gamma}(A) \leq s$,
(2) a $\mathcal{T}^{\mu\gamma}$-*fuzzy* (r,s)-*closed* set if $\mathcal{T}^{\mu}(A^c) \geq r$ and $\mathcal{T}^{\gamma}(A^c) \leq s$.

Let $(X, \mathcal{T}^{\mu}, \mathcal{T}^{\gamma})$ be an intuitionistic fuzzy topological space in Šostak's sense. For each $(r,s) \in I \otimes I$ and for each $A \in I(X)$, the $\mathcal{T}^{\mu\gamma}$-*fuzzy* (r,s)-*closure* is defined by

$$\mathcal{T}^{\mu\gamma}\text{-cl}(A,(r,s)) = \bigcap\{B \in I(X) \mid A \subseteq B, B \text{ is } \mathcal{T}^{\mu\gamma}\text{-fuzzy } (r,s)\text{-closed}\}$$

and the $\mathcal{T}^{\mu\gamma}$-*fuzzy* (r,s)-*interior* is defined by

$$\mathcal{T}^{\mu\gamma}\text{-int}(A,(r,s)) = \bigcup\{B \in I(X) \mid A \supseteq B, B \text{ is } \mathcal{T}^{\mu\gamma}\text{-fuzzy } (r,s)\text{-open}\}.$$

Lemma 2.3. [9] For an intuitionistic fuzzy set A in an intuitionistic fuzzy topological space in Šostak's sense $(X, \mathcal{T}^{\mu}, \mathcal{T}^{\gamma})$ and $(r,s) \in I \otimes I$, we have:

(1) $\mathcal{T}^{\mu\gamma}\text{-cl}(A,(r,s))^c = \mathcal{T}^{\mu\gamma}\text{-int}(A^c,(r,s))$.
(2) $\mathcal{T}^{\mu\gamma}\text{-int}(A,(r,s))^c = \mathcal{T}^{\mu\gamma}\text{-cl}(A^c,(r,s))$.

A system $(X, \mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$ consisting of a set X with two intuitionistic fuzzy topologies in Šostak's sense $\mathcal{T}^{\mu\gamma}$ and $\mathcal{U}^{\mu\gamma}$ on X is called a *double bitopological space.*

Definition 2.4. [10] Let A be an intuitionistic fuzzy set of a double bitopological space $(X, \mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$ and $(r,s),(u,v) \in I \otimes I$. Then A is said to be

(1) $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-*double* $(r,s)(u,v)$-*preopen* if
$A \subseteq \mathcal{T}^{\mu\gamma}\text{-int}(\mathcal{U}^{\mu\gamma}\text{-cl}(A,u,v),r,s)$,
(2) $(\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})$-*double* $(u,v)(r,s)$-*preopen* if
$A \subseteq \mathcal{U}^{\mu\gamma}\text{-int}(\mathcal{T}^{\mu\gamma}\text{-cl}(A,r,s),u,v)$,
(3) $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-*double* $(r,s)(u,v)$-*preclosed* if
$A \supseteq \mathcal{T}^{\mu\gamma}\text{-cl}((\mathcal{U}^{\mu\gamma}\text{-int}(A,u,v),r,s)$,
(4) $(\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})$-*double* $(u,v)(r,s)$-*preclosed* if
$A \supseteq \mathcal{U}^{\mu\gamma}\text{-cl}((\mathcal{T}^{\mu\gamma}\text{-int}(A,r,s),u,v)$.

3. Double pairwise $(r,s)(u,v)$-precontinuous mappings

DEFINITION 3.1. Let $(X,\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$ be a double bitopological space and $(r,s),(u,v) \in I \otimes I$. For each $A \in I(X)$, the $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-*double* $(r,s)(u,v)$-*preclosure* is defined by

$$(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})\text{-dpcl}(A,(r,s),(u,v))$$
$$= \bigcap\{B \in I(X) \mid B \supseteq A, B \text{ is } (\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})\text{-double } (r,s)(u,v)\text{-preclosed}\}$$

and the $(\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})$-*double* $(u,v)(r,s)$-*preclosure* is defined by

$$(\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})\text{-dpcl}(A,(u,v),(r,s))$$
$$= \bigcap\{B \in I(X) \mid B \supseteq A, B \text{ is } (\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})\text{-double } (u,v)(r,s)\text{-preclosed}\}.$$

DEFINITION 3.2. Let $(X,\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$ be a double bitopological space and $(r,s),(u,v) \in I \otimes I$. For each $A \in I(X)$, the $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-*double* $(r,s)(u,v)$-*preinterior* is defined by

$$(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})\text{-dpint}(A,(r,s),(u,v))$$
$$= \bigcup\{B \in I(X) \mid B \subseteq A, B \text{ is } (\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})\text{-double } (r,s)(u,v)\text{-preopen}\}$$

and the $(\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})$-*double* $(u,v)(r,s)$-*preinterior* is defined by

$$(\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})\text{-dpint}(A,(u,v),(r,s))$$
$$= \bigcup\{B \in I(X) \mid B \subseteq A, B \text{ is } (\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})\text{-double } (u,v)(r,s)\text{-preopen}\}.$$

Obviously, $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})\text{-dpcl}(A,(r,s),(u,v))$ is the smallest $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-double $(r,s)(u,v)$-preclosed set which contains A and $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})\text{-dpcl}(A,(r,s),(u,v)) = A$ for any $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-double $(r,s)(u,v)$-preclosed set A. Also $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})\text{-dpint}(A,(r,s),(u,v))$ is the greatest $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-double $(r,s)(u,v)$-preopen set which is contained A and

$(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-dpint$(A,(r,s),(u,v)) = A$ for any $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-double $(r,s)(u,v)$-preopen set A. Moreover, we have

$$\begin{aligned}\mathcal{T}^{\mu\gamma}\text{-int}(A,(r,s)) &\subseteq (\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})\text{-dpint}(A,(r,s),(u,v))\\ &\subseteq A\\ &\subseteq (\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})\text{-dpcl}(A,(r,s),(u,v))\\ &\subseteq \mathcal{T}^{\mu\gamma}\text{-cl}(A,(r,s)).\end{aligned}$$

Also, we have the following results:

(1) $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-dpcl$(0_\sim,(r,s),(u,v)) = 0_\sim$ and $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-dpcl$(1_\sim,(r,s),(u,v)) = 1_\sim$.
(2) $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-dpcl$(A,(r,s),(u,v)) \supseteq A$.
(3) $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-dpcl$(A\cup B,(r,s),(u,v))$ $\supseteq (\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-dpcl$(A,(r,s),(u,v))\cup(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-dpcl$(B,(r,s),(u,v))$.
(4) $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-dpcl$((\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-dpcl$(A,(r,s),(u,v)),(r,s),(u,v))$ $= (\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-dpcl$(A,(r,s),(u,v))$.
(5) $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-dpint$(0_\sim,(r,s),(u,v)) = 0_\sim$ and $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-dpint$(1_\sim,(r,s),(u,v)) = 1_\sim$.
(6) $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-dpint$(A,(r,s),(u,v)) \subseteq A$.
(7) $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-dpint$(A\cap B,(r,s),(u,v))$ $\subseteq (\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-dpint$(A,(r,s),(u,v))\cap(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-dpint$(B,(r,s),(u,v))$.
(8) $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-dpint$((\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-dpint$(A,(r,s),(u,v)),(r,s),(u,v))$ $= (\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-dpint$(A,(r,s),(u,v))$.

Theorem 3.3. Let A be an intuitionistic fuzzy set of a double bitopological space $(X,\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$ and $(r,s),(u,v)\in I\otimes I$. Then we have

(1) $((\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})\text{-dpint}(A,(r,s),(u,v)))^c = (\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})\text{-dpcl}(A^c,(r,s),(u,v))$.
(2) $((\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})\text{-dpcl}(A,(r,s),(u,v)))^c = (\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})\text{-dpint}(A^c,(r,s),(u,v))$.

Proof. (1) Since $A^c \subseteq (\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-dpcl$(A^c,(r,s),(u,v))$ and $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-dpcl$(A^c,(r,s),(u,v))$ is a $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-double $(r,s)(u,v)$-preclosed set, we have $((\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})\text{-dpcl}(A^c,(r,s),(u,v)))^c \subseteq A$ and $((\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})\text{-dpcl}(A^c,(r,s),(u,v)))^c$ is a $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-double $(r,s)(u,v)$-preopen set. Thus

$$\begin{aligned}&((\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})\text{-dpcl}(A^c,(r,s),(u,v)))^c\\ &= (\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})\text{-dpint}(((\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})\text{-dpcl}(A^c,(r,s),(u,v)))^c,(r,s),(u,v))\\ &\subseteq (\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})\text{-dpint}(A,(r,s),(u,v))\end{aligned}$$

and hence

$$((\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})\text{-dpint}(A,(r,s),(u,v)))^c \subseteq (\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})\text{-dpcl}(A^c,(r,s),(u,v)).$$

Conversely, since $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-dpint$(A,(r,s),(u,v)) \subseteq A$ and $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-dpint$(A,(r,s),(u,v))$ is a $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-double $(r,s)(u,v)$-preopen set, we have $A^c \subseteq ((\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})\text{-dpint}(A,(r,s),(u,v)))^c$ and $((\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})\text{-dpint}(A,(r,s),(u,v)))^c$ is a $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-double $(r,s)(u,v)$-preclosed set. Thus

$$\begin{aligned}&(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})\text{-dpcl}(A^c,(r,s),(u,v))\\ &\subseteq (\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})\text{-dpcl}(((\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})\text{-dpint}(A,(r,s),(u,v)))^c,(r,s),(u,v))\\ &= ((\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})\text{-dpint}(A,(r,s),(u,v)))^c.\end{aligned}$$

(2) Similar to (1) □

COROLLARY 3.4. Let A be an intuitionistic fuzzy set of a double bitopological space $(X,\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$ and $(r,s),(u,v) \in I \otimes I$. Then we have

(1) $((\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})\text{-dpint}(A,(u,v),(r,s)))^c = (\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})\text{-dpcl}(A^c,(u,v),(r,s))$.
(2) $((\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})\text{-dpcl}(A,(u,v),(r,s)))^c = (\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})\text{-dpint}(A^c,(u,v),(r,s))$.

DEFINITION 3.5. Let $f : (X,\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma}) \to (Y,\mathcal{V}^{\mu\gamma},\mathcal{W}^{\mu\gamma})$ be a mapping from a double bitopological space X to a double bitopological space Y and $(r,s),(u,v) \in I \otimes I$. Then f is called

(1) double pairwise $(r,s)(u,v)$-continuous if the induced mapping $f : (X,\mathcal{T}^{\mu\gamma}) \to (Y,\mathcal{V}^{\mu\gamma})$ is fuzzy (r,s)-continuous and the induced mapping $f : (X,\mathcal{U}^{\mu\gamma}) \to (Y,\mathcal{W}^{\mu\gamma})$ is fuzzy continuous,
(2) double pairwise $(r,s)(u,v)$-precontinuous if $f^{-1}(A)$ is a $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-double $(r,s)(u,v)$-preopen set of X for each $\mathcal{V}^{\mu\gamma}$-fuzzy (r,s)-open set A of Y and $f^{-1}(B)$ is a $(\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})$-double $(u,v)(r,s)$-preopen set of X for each $\mathcal{W}^{\mu\gamma}$-fuzzy (u,v)-open set B of Y, or equivalently, $f^{-1}(A)$ is a $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-double $(r,s)(u,v)$-preclosed set of X for each $\mathcal{V}^{\mu\gamma}$-fuzzy (r,s)-closed set A of Y and $f^{-1}(B)$ is a $(\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})$-double $(u,v)(r,s)$-preclosed set of X for each $\mathcal{W}^{\mu\gamma}$-fuzzy (u,v)-closed set B of Y.

It is obvious that every double pairwise $(r,s)(u,v)$-continuous mapping is a double pairwise $(r,s)(u,v)$-precontinuous mapping but the converse need not be true which is shown by the following example.

EXAMPLE 3.6. Let $X = \{x,y\}$ and let A_1, A_2, A_3 and A_4 be intuitionistic fuzzy sets of X defined as

$$A_1(x) = (0.2, 0.4), \quad A_1(y) = (0.6, 0.3);$$

$$A_2(x) = (0.4, 0.3), \quad A_2(y) = (0.7, 0.1);$$

$$A_3(x) = (0.5, 0.2), \quad A_3(y) = (0.1, 0.8);$$

and

$$A_4(x) = (0.5, 0.3), \quad A_4(y) = (0.2, 0.4).$$

Define $\mathcal{T}^{\mu\gamma} : I(X) \to I \otimes I$ and $\mathcal{U}^{\mu\gamma} : I(X) \to I \otimes I$ by

$$\mathcal{T}^{\mu\gamma}(A) = (\mathcal{T}^{\mu}(A), \mathcal{T}^{\gamma}(A)) = \begin{cases} (1, 0) & \text{if } A = 0_\sim, 1_\sim, \\ (\frac{1}{2}, \frac{1}{5}) & \text{if } A = A_1, \\ (0, 1) & \text{otherwise;} \end{cases}$$

and

$$\mathcal{U}^{\mu\gamma}(A) = (\mathcal{U}^{\mu}(A), \mathcal{U}^{\gamma}(A)) = \begin{cases} (1, 0) & \text{if } A = 0_\sim, 1_\sim, \\ (\frac{1}{3}, \frac{1}{4}) & \text{if } A = A_2, \\ (0, 1) & \text{otherwise.} \end{cases}$$

Then clearly $(X, \mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$ is a double bitopological space on X. Define $\mathcal{V}^{\mu\gamma} : I(X) \to I \otimes I$ and $\mathcal{W}^{\mu\gamma} : I(X) \to I \otimes I$ by

$$\mathcal{V}^{\mu\gamma}(A) = (\mathcal{V}^{\mu}(A), \mathcal{V}^{\gamma}(A)) = \begin{cases} (1, 0) & \text{if } A = 0_\sim, 1_\sim, \\ (\frac{1}{2}, \frac{1}{5}) & \text{if } A = A_3, \\ (0, 1) & \text{otherwise;} \end{cases}$$

and

$$\mathcal{W}^{\mu\gamma}(A) = (\mathcal{W}^{\mu}(A), \mathcal{W}^{\gamma}(A)) = \begin{cases} (1, 0) & \text{if } A = 0_\sim, 1_\sim, \\ (\frac{1}{3}, \frac{1}{4}) & \text{if } A = A_4, \\ (0, 1) & \text{otherwise.} \end{cases}$$

Then clearly $(X, \mathcal{V}^{\mu\gamma}, \mathcal{W}^{\mu\gamma})$ is a double bitopological space on X. Consider the identity mapping $1_X : (X, \mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma}) \to (X, \mathcal{V}^{\mu\gamma}, \mathcal{W}^{\mu\gamma})$. Then it is a double pairwise $(\frac{1}{2}, \frac{1}{5})(\frac{1}{3}, \frac{1}{4})$-precontinuous mapping which is not a double pairwise $(\frac{1}{2}, \frac{1}{5})(\frac{1}{3}, \frac{1}{4})$-continuous mapping.

Remark 3.7. That double pairwise $(r, s)(u, v)$-precontinuous and double $(r, s)(u, v)$-semicontinuous are independent notions is shown by the following example.

Example 3.8. Let $X = \{x, y\}$ and let A_1, A_2, A_3, A_4, A_5 and A_6 be intuitionistic fuzzy sets of X defined as

$$A_1(x) = (0.2, 0.7), \quad A_1(y) = (0.5, 0.3);$$

$$A_2(x) = (0.5, 0.4), \quad A_2(y) = (0.2, 0.6);$$

$$A_3(x) = (0.5, 0.3), \quad A_3(y) = (0.4, 0.2);$$

$$A_4(x) = (0.8, 0.1), \quad A_4(y) = (0.1, 0.7);$$

$$A_5(x) = (0.3, 0.6), \quad A_5(y) = (0.5, 0.2);$$

and

$$A_6(x) = (0.6, 0.2), \quad A_6(y) = (0.2, 0.5).$$

Define $\mathcal{T}^{\mu\gamma} : I(X) \to I \otimes I$ and $\mathcal{U}^{\mu\gamma} : I(X) \to I \otimes I$ by

$$\mathcal{T}^{\mu\gamma}(A) = (\mathcal{T}^{\mu}(A), \mathcal{T}^{\gamma}(A)) = \begin{cases} (1,0) & \text{if } A = 0_\sim, 1_\sim, \\ (\frac{1}{2}, \frac{1}{5}) & \text{if } A = A_1, \\ (0,1) & \text{otherwise;} \end{cases}$$

and

$$\mathcal{U}^{\mu\gamma}(A) = (\mathcal{U}^{\mu}(A), \mathcal{U}^{\gamma}(A)) = \begin{cases} (1,0) & \text{if } A = 0_\sim, 1_\sim, \\ (\frac{1}{3}, \frac{1}{4}) & \text{if } A = A_2, \\ (0,1) & \text{otherwise.} \end{cases}$$

Define $\mathcal{V}^{\mu\gamma} : I(X) \to I \otimes I$ and $\mathcal{W}^{\mu\gamma} : I(X) \to I \otimes I$ by

$$\mathcal{V}^{\mu\gamma}(A) = (\mathcal{V}^{\mu}(A), \mathcal{V}^{\gamma}(A)) = \begin{cases} (1,0) & \text{if } A = 0_\sim, 1_\sim, \\ (\frac{1}{2}, \frac{1}{5}) & \text{if } A = A_3, \\ (0,1) & \text{otherwise;} \end{cases}$$

and

$$\mathcal{W}^{\mu\gamma}(A) = (\mathcal{W}^{\mu}(A), \mathcal{W}^{\gamma}(A)) = \begin{cases} (1,0) & \text{if } A = 0_\sim, 1_\sim, \\ (\frac{1}{3}, \frac{1}{4}) & \text{if } A = A_4, \\ (0,1) & \text{otherwise.} \end{cases}$$

Define $\mathcal{F}^{\mu\gamma} : I(X) \to I \otimes I$ and $\mathcal{G}^{\mu\gamma} : I(X) \to I \otimes I$ by

$$\mathcal{F}^{\mu\gamma}(A) = (\mathcal{F}^{\mu}(A), \mathcal{F}^{\gamma}(A)) = \begin{cases} (1,0) & \text{if } A = 0_\sim, 1_\sim, \\ (\frac{1}{2}, \frac{1}{5}) & \text{if } A = A_5, \\ (0,1) & \text{otherwise;} \end{cases}$$

and

$$\mathcal{G}^{\mu\gamma}(A) = (\mathcal{G}^{\mu}(A), \mathcal{G}^{\gamma}(A)) = \begin{cases} (1,0) & \text{if } A = 0_\sim, 1_\sim, \\ (\frac{1}{3}, \frac{1}{4}) & \text{if } A = A_6, \\ (0,1) & \text{otherwise.} \end{cases}$$

Then clearly $(X, \mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$, $(X, \mathcal{V}^{\mu\gamma}, \mathcal{W}^{\mu\gamma})$ and $(X, \mathcal{F}^{\mu\gamma}, \mathcal{G}^{\mu\gamma})$ are double bitopological spaces on X. The identity mapping $1_X : (X, \mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma}) \to (X, \mathcal{V}^{\mu\gamma}, \mathcal{W}^{\mu\gamma})$ is a double pairwise $(\frac{1}{2}, \frac{1}{5})(\frac{1}{3}, \frac{1}{4})$-precontinuous mapping which is not a double pairwise $(\frac{1}{2}, \frac{1}{5})(\frac{1}{3}, \frac{1}{4})$-semicontinuous mapping. Also, the identity mapping $1_X : (X, \mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma}) \to (X, \mathcal{F}^{\mu\gamma}, \mathcal{G}^{\mu\gamma})$ is a double pairwise $(\frac{1}{2}, \frac{1}{5})(\frac{1}{3}, \frac{1}{4})$-semicontinuous mapping which is not a double pairwise $(\frac{1}{2}, \frac{1}{5})(\frac{1}{3}, \frac{1}{4})$-precontinuous mapping.

THEOREM 3.9. Let $f:(X,\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})\to(Y,\mathcal{V}^{\mu\gamma},\mathcal{W}^{\mu\gamma})$ be a mapping and $(r,s),(u,v)\in I\otimes I$. Then the following statements are equivalent:

(1) f is a double pairwise $(r,s)(u,v)$-precontinuous mapping.

(2) For each intuitionistic fuzzy set C of X,

$$f((\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})\text{-dpcl}(C,(r,s),(u,v)))\subseteq\mathcal{V}^{\mu\gamma}\text{-cl}(f(C),(r,s))$$

and

$$f((\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})\text{-dpcl}(C,(u,v),(r,s)))\subseteq\mathcal{W}^{\mu\gamma}\text{-cl}(f(C),(u,v)).$$

(3) For each intuitionistic fuzzy set A of Y,

$$(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})\text{-dpcl}(f^{-1}(A),(r,s),(u,v))\subseteq f^{-1}(\mathcal{V}^{\mu\gamma}\text{-cl}(A,(r,s)))$$

and

$$(\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})\text{-dpcl}(f^{-1}(A),(u,v),(r,s))\subseteq f^{-1}(\mathcal{W}^{\mu\gamma}\text{-cl}(A,(u,v))).$$

(4) For each intuitionistic fuzzy set A of Y,

$$f^{-1}(\mathcal{V}^{\mu\gamma}\text{-int}(A,(r,s)))\subseteq(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})\text{-dpint}(f^{-1}(A),(r,s),(u,v))$$

and

$$f^{-1}(\mathcal{W}^{\mu\gamma}\text{-int}(A,(u,v)))\subseteq(\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})\text{-dpint}(f^{-1}(A),(u,v),(r,s)).$$

Proof. (1) $\Rightarrow$ (2) Let C be any intuitionistic fuzzy set of X. Then $f(C)$ is an intuitionistic fuzzy set of Y, and hence $\mathcal{V}^{\mu\gamma}$-cl$(f(C),(r,s))$ is a $\mathcal{V}^{\mu\gamma}$-fuzzy (r,s)-closed set and $\mathcal{W}^{\mu\gamma}$-cl$(f(C),(u,v))$ is a $\mathcal{W}^{\mu\gamma}$-fuzzy (u,v)-closed set of Y. Since f is a double pairwise $(r,s)(u,v)$-precontinuous mapping, $f^{-1}(\mathcal{V}^{\mu\gamma}\text{-cl}(f(C),(r,s)))$ is a $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-double $(r,s)(u,v)$-preclosed set and $f^{-1}(\mathcal{W}^{\mu\gamma}\text{-cl}(f(C),(u,v)))$ is a $(\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})$-double $(u,v)(r,s)$-preclosed set of X. Also

$$C\subseteq f^{-1}f(C)\subseteq f^{-1}(\mathcal{V}^{\mu\gamma}\text{-cl}(f(C),(r,s)))$$

and

$$C\subseteq f^{-1}f(C)\subseteq f^{-1}(\mathcal{W}^{\mu\gamma}\text{-cl}(f(C),(u,v))).$$

Thus

$$\begin{aligned}&(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})\text{-dpcl}(C,(r,s),(u,v))\\&\subseteq(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})\text{-dpcl}(f^{-1}(\mathcal{V}^{\mu\gamma}\text{-cl}(f(C),(r,s))),(r,s),(u,v))\\&=f^{-1}(\mathcal{V}^{\mu\gamma}\text{-cl}(f(C),(r,s)))\end{aligned}$$

and

$$\begin{aligned}&(\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})\text{-dpcl}(C,(u,v),(r,s))\\&\subseteq(\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})\text{-dpcl}(f^{-1}(\mathcal{W}^{\mu\gamma}\text{-cl}(f(C),(u,v))),(u,v),(r,s))\\&=f^{-1}(\mathcal{W}^{\mu\gamma}\text{-cl}(f(C),(u,v))).\end{aligned}$$

Hence

$$\begin{aligned}f((\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})\text{-dpcl}(C,(r,s),(u,v)))&\subseteq ff^{-1}(\mathcal{V}^{\mu\gamma}\text{-cl}(f(C),(r,s)))\\&\subseteq\mathcal{V}^{\mu\gamma}\text{-cl}(f(C),(r,s))\end{aligned}$$

and

$$\begin{aligned}f((\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})\text{-dpcl}(C,(u,v),(r,s)))&\subseteq ff^{-1}(\mathcal{W}^{\mu\gamma}\text{-cl}(f(C),(u,v)))\\&\subseteq\mathcal{W}^{\mu\gamma}\text{-cl}(f(C),(u,v)).\end{aligned}$$

(2) $\Rightarrow$ (3) Let A be any intuitionistic fuzzy set of Y. Then $f^{-1}(A)$ is an intuitionistic fuzzy set of X. By (2) we have,

$$\begin{aligned}f((\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})\text{-dpcl}(f^{-1}(A),(r,s),(u,v)))&\subseteq\mathcal{V}^{\mu\gamma}\text{-cl}(ff^{-1}(A),(r,s))\\&\subseteq\mathcal{V}^{\mu\gamma}\text{-cl}(A,(r,s))\end{aligned}$$

and

$$\begin{aligned}f((\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})\text{-dpcl}(f^{-1}(A),(u,v),(r,s)))&\subseteq\mathcal{W}^{\mu\gamma}\text{-cl}(ff^{-1}(A),(u,v))\\&\subseteq\mathcal{W}^{\mu\gamma}\text{-cl}(A,(u,v)).\end{aligned}$$

Thus

$$\begin{aligned}&(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})\text{-dpcl}(f^{-1}(A),(r,s),(u,v))\\&\subseteq f^{-1}f((\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})\text{-dpcl}(f^{-1}(A),(r,s),(u,v)))\\&\subseteq f^{-1}(\mathcal{V}^{\mu\gamma}\text{-cl}(A,(r,s)))\end{aligned}$$

and

$$\begin{aligned}&(\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})\text{-dpcl}(f^{-1}(A),(u,v),(r,s))\\&\subseteq f^{-1}f((\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})\text{-dpcl}(f^{-1}(A),(u,v),(r,s)))\\&\subseteq f^{-1}(\mathcal{W}^{\mu\gamma}\text{-cl}(A,(u,v))).\end{aligned}$$

(3) $\Rightarrow$ (4) Let A be any intuitionistic fuzzy set of Y. Then

$$(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})\text{-dpcl}(f^{-1}(A)^c,(r,s),(u,v))\subseteq f^{-1}(\mathcal{V}^{\mu\gamma}\text{-cl}(A^c,(r,s)))$$

and

$$(\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})\text{-dpcl}(f^{-1}(A)^c,(u,v),(r,s))\subseteq f^{-1}(\mathcal{W}^{\mu\gamma}\text{-cl}(A^c,(u,v))).$$

By Theorem 3.3 and Corollary 3.4,

$$\begin{aligned} f^{-1}(\mathcal{V}^{\mu\gamma}\text{-int}(A,(r,s))) &= (f^{-1}(\mathcal{V}^{\mu\gamma}\text{-cl}(A^c,(r,s))))^c \\ &\subseteq ((\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})\text{-dpcl}(f^{-1}(A)^c,(r,s),(u,v)))^c \\ &= (\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})\text{-dpint}(f^{-1}(A),(r,s),(u,v)) \end{aligned}$$

and

$$\begin{aligned} f^{-1}(\mathcal{W}^{\mu\gamma}\text{-int}(A,(u,v))) &= (f^{-1}(\mathcal{W}^{\mu\gamma}\text{-cl}(A^c,(u,v))))^c \\ &\subseteq ((\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})\text{-dpcl}(f^{-1}(A)^c,(u,v),(r,s)))^c \\ &= (\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})\text{-dpint}(f^{-1}(A),(u,v),(r,s)). \end{aligned}$$

(4) $\Rightarrow$ (1) Let A be any $\mathcal{V}^{\mu\gamma}$-fuzzy (r,s)-open set and B any $\mathcal{W}^{\mu\gamma}$-fuzzy (u,v)-open set of Y. Then $\mathcal{V}^{\mu\gamma}\text{-int}(A,(r,s)) = A$ and $\mathcal{W}^{\mu\gamma}\text{-int}(B,(u,v)) = B$. Thus

$$\begin{aligned} f^{-1}(A) &= f^{-1}(\mathcal{V}^{\mu\gamma}\text{-int}(A,(r,s))) \\ &\subseteq (\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})\text{-dpint}(f^{-1}(A),(r,s),(u,v)) \\ &\subseteq f^{-1}(A) \end{aligned}$$

and

$$\begin{aligned} f^{-1}(B) &= f^{-1}(\mathcal{W}^{\mu\gamma}\text{-int}(B,(u,v))) \\ &\subseteq (\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})\text{-dpint}(f^{-1}(B),(u,v),(r,s)) \\ &\subseteq f^{-1}(B). \end{aligned}$$

So

$$f^{-1}(A) = (\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})\text{-dpint}(f^{-1}(A),(r,s),(u,v))$$

and

$$f^{-1}(B) = (\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})\text{-dpint}(f^{-1}(B),(u,v),(r,s)).$$

Hence $f^{-1}(A)$ is a $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-double $(r,s)(u,v)$-preopen set and $f^{-1}(B)$ is a $(\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})$-double $(u,v)(r,s)$-preopen set of X. Therefore f is a double pairwise $(r,s)(u,v)$-precontinuous mapping. □

Theorem 3.10. Let $f : (X,\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma}) \to (Y,\mathcal{V}^{\mu\gamma},\mathcal{W}^{\mu\gamma})$ be a bijection and $(r,s),(u,v) \in I \otimes I$. Then f is a double pairwise $(r,s)(u,v)$-precontinuous mapping if and only if for each intuitionistic fuzzy set C of X,

$$\mathcal{V}^{\mu\gamma}\text{-int}(f(C),(r,s)) \subseteq f((\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})\text{-dpint}(C,(r,s),(u,v)))$$

and

$$\mathcal{W}^{\mu\gamma}\text{-int}(f(C),(u,v)) \subseteq f((\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})\text{-dpint}(C,(u,v),(r,s))).$$

Proof. Let C be any intuitionistic fuzzy set of X. Since f is one-to-one,

$$\begin{aligned} f^{-1}(\mathcal{V}^{\mu\gamma}\text{-int}(f(C),(r,s))) &\subseteq (\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})\text{-dpint}(f^{-1}f(C),(r,s),(u,v)) \\ &= (\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})\text{-dpint}(C,(r,s),(u,v)) \end{aligned}$$

and

$$\begin{aligned} f^{-1}(\mathcal{W}^{\mu\gamma}\text{-int}(f(C),(u,v))) &\subseteq (\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})\text{-dpint}(f^{-1}f(C),(u,v),(r,s)) \\ &= (\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})\text{-dpint}(C,(u,v),(r,s)). \end{aligned}$$

Since f is onto, we have

$$\begin{aligned} \mathcal{V}^{\mu\gamma}\text{-int}(f(C),(r,s)) &= ff^{-1}(\mathcal{V}^{\mu\gamma}\text{-int}(f(C),(r,s))) \\ &\subseteq f((\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})\text{-dpint}(C,(r,s),(u,v))) \end{aligned}$$

and

$$\begin{aligned} \mathcal{W}^{\mu\gamma}\text{-int}(f(C),(u,v)) &= ff^{-1}(\mathcal{W}^{\mu\gamma}\text{-int}(f(C),(u,v))) \\ &\subseteq f((\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})\text{-dpint}(C,(u,v),(r,s))). \end{aligned}$$

Conversely, let A be an intuitionistic fuzzy set of Y. Since f is onto,

$$\begin{aligned} \mathcal{V}^{\mu\gamma}\text{-int}(A,(r,s)) &= \mathcal{V}^{\mu\gamma}\text{-int}(ff^{-1}(A),(r,s)) \\ &\subseteq f((\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})\text{-dpint}(f^{-1}(A),(r,s),(u,v))) \end{aligned}$$

and

$$\begin{aligned} \mathcal{W}^{\mu\gamma}\text{-int}(A,(u,v)) &= \mathcal{W}^{\mu\gamma}\text{-int}(ff^{-1}(A),(u,v)) \\ &\subseteq f((\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})\text{-dpint}(f^{-1}(A),(u,v),(r,s))). \end{aligned}$$

Since f is one-to-one, we have

$$\begin{aligned} f^{-1}(\mathcal{V}^{\mu\gamma}\text{-int}(A,(r,s))) &\subseteq f^{-1}f((\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})\text{-dpint}(f^{-1}(A),(r,s),(u,v))) \\ &= (\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})\text{-dpint}(f^{-1}(A),(r,s),(u,v)) \end{aligned}$$

and

$$\begin{aligned} f^{-1}(\mathcal{W}^{\mu\gamma}\text{-int}(A,(u,v))) &\subseteq f^{-1}f((\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})\text{-dpint}(f^{-1}(A),(u,v),(r,s))) \\ &= (\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})\text{-dpint}(f^{-1}(A),(u,v),(r,s)). \end{aligned}$$

Hence the theorem follows. □

References

[1] K. T. Atanassov, *Intuitionistic fuzzy sets*, Fuzzy Sets and Systems **20** (1986), 87-96.
[2] C. L. Chang, *Fuzzy topological spaces*, J. Math. Anal. Appl. **24** (1968), 182–190.
[3] K. C. Chattopadhyay, R. N. Hazra, and S. K. Samanta, *Gradation of openness : Fuzzy topology*, Fuzzy Sets and Systems **49** (1992), 237-242.
[4] D. Çoker, *An introduction to intuitionistic fuzzy topological spaces*, Fuzzy Sets and Systems **88** (1997), 81-89.
[5] D. Çoker and M. Demirci, *An introduction to intuitionistic fuzzy topological spaces in Šostak's sense*, BUSEFAL **67** (1996), 67-76.
[6] D. Çoker and A. Haydar Eş, *On fuzzy compactness in intuitionistic fuzzy topological spaces*, J. Fuzzy Math. **3** (1995), 899-909.
[7] H. Gürçay, D. Çoker, and A. Haydar Eş, *On fuzzy continuity in intuitionistic fuzzy topological spaces*, J. Fuzzy Math. **5** (1997), 365-378.
[8] A. Kandil, *Biproximities and fuzzy bitopological spaces*, Simon Stevin **63** (1989), 45-66.
[9] E. P. Lee and S. O. Lee, *Double semiopen sets on double bitopological spaces*, Journal of the Chungcheong Mathematical Society **26** (2013), 691-702.
[10] E. P. Lee and S. O. Lee, *Double $(r,s)(u,v)$-preopen sets*, Journal of the Chungcheong Mathematical Society **29** (2016), 13-22.
[11] A. A. Ramadan, *Smooth topological spaces*, Fuzzy Sets and Systems **48** (1992), 371-375.
[12] A. P. Šostak, *On a fuzzy topological structure*, Suppl. Rend. Circ. Matem. Janos Palermo, Sr. II **11** (1985), 89-103.

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